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NEW CONCEPTS IN THE THEORY OF TOPOLOGICAL SPACE—SUPERCONDENSED SET, SUBCONDENSED SET, AND CONDENSED SET.

YOSHINORI ISOMICHI

A set which satisfies $A^{ai} = A^{i}$ ($A^{ia} = A^{a}$) is named a supercondensed (subcondensed) set. A supercondensed and subcondensed set is named a condensed set. Theorems about these new concepts are discussed in this paper.

Sets of the topological space are classified into three classes by comparing A^{ai} and A^{ia}.

The theory of topological space is usually constructed upon the axioms of *neighborhood*. One defines "open set", "closed set", "open kernel", and "closure" using the concept of neighborhood. Furthermore one introduces the concept of *accumulation point* and defines "perfect set", "set which is dense in itself", and "isolated set".

Accumulation points are classified by their cardinal numbers of their neighborhoods. In particular, a point every neighborhood of which contains points of the set more than $\aleph_1$ is called a "condensation point". (A set which coincides with its all condensation points is called a condensed set, hitherto. But in this paper the terminology "condensed set" is used for a different meaning.)

In this paper we define the following new concepts; *supercondensed set*, *subcondensed set*, and *condensed set*, not using the concepts of accumulation point and cardinal number, but using the concepts of open kernel and closure only. So it may be advocated that these new concepts are fairly basic ones. Further we introduce concepts of "*border of a set*" (which is different from "boundary of a set") and discuss about the relations between these new concepts. Furthermore we classify sets of the topological space into three classes.

In the following discussion we assume that A, B, C, etc. are the subsets of the topological space X. The open kernel of a subset A is denoted by A^{i}, closure and complemet of A are denoted by A^{a}, A^{c}, respectively.

LEMMA 1. (Kuratowski Theorem 6). *For any A of X, the following relations hold.*

$$A^{aiai} = A^{ai}$$
$$A^{iaia} = A^{ia}$$