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SEPARATING CERTAIN PLANE-LIKE SPACES BY PEANO CONTINUA

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Suppose S is a connected, locally connected, complete Moore space having no cut point and in which the Jordan Curve Theorem holds. Thus, suppose S satisfies R. L. Moore's Axioms 0, 1-4. Certain extensions and applications of earlier results of the author are established. In particular, modified forms of the Torhorst theorem and a plane theorem of R. L. Moore are shown to hold in the space S, two theorems concerning the separation of S by compact dendrons are extended to Peano continua and another to certain Menger regular curves. Finally, a general method of constructing certain pathological spaces is given.

In [1], Gref showed that if the space S is metrizable (which does not follow from the axioms indicated), which implies separability [5], [7], and no arc separates S, then many of the known separation theorems of plane topology can be established for the space S. His method of argument, however, is not valid in the present setting and indeed, many of the theorems he establishes are not true for the space here considered.

Extensive use is made of the results and terminology of [6] and [2]. The term "continuous curve" is to mean any connected, locally connected, closed point set, whether compact or not. A point E of a continuous curve M is an endpoint of M if and only if E is an endpoint of every arc in M containing E. If M is compact, this is equivalent to the definition in [8, p. 64].

THEOREM 1 (Modified Torhorst Theorem). *If U is a complementary domain of a Peano continuum M, there exists a Menger regular curve in M which is an irreducible continuum about the boundary of U.*

Proof. Let ω denote a point of U and for each simple closed curve J in M, let D_J denote the interior of J with respect to ω; that is, let D_J denote the complementary domain of J not containing ω. If M contains no simple closed curve, the stated conclusion follows immediately from the fact that M is a Menger regular curve and thus is hereditarily locally connected [8, p. 99]. Let M' denote the set of all points of M not belonging to D_J for any simple closed curve J in M. Clearly, M' is a nonempty closed subset of the compact point set M. Suppose D is a proper domain with respect to M