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MULTIPLIERS OF QUOTIENTS OF L_1

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Let G be a locally compact abelian group with dual Γ. The multiplier problem for $L_1(G)$ has a well known and easy solution, while the corresponding problem for its ideals is subtle. So far as we have been able to determine, the problem for quotient algebras of $L_1(G)$ has not received attention. The purpose of this note is to point that out and to give a condition on the quotient, which ensures that the simplest possible answer holds, and an example, which shows that in general that answer is false.

1. The multiplier problem for ideals is treated in [2]. We denote by $M(G)$ the finite, regular, complex valued measures on G, by $A(\Gamma)$ the Fourier transforms of functions in $L_1(G)$, by $B(\Gamma)$ the Fourier-Stieltjes transforms of measures in $M(G)$ and by $f \mid E$ the restriction of a function f to the set E. Finally $A(E) = A(\Gamma) \mid E$ and $B(E) = B(\Gamma) \mid E$. The problem can be stated in a somewhat greater generality as follows:

Let E be a subset of Γ and φ a (necessarily bounded and continuous) function on E, for which

$$\varphi A(E) \subset A(E) \ . \tag{1.1}$$

Under what conditions on E does this imply that $\varphi \in B(E)$? If Γ is compact $\hat{f} \equiv 1$ belongs to $A(\Gamma)$ and it follows that (1.1) implies $\varphi \in A(E) (= B(E))$ for every E. In the present section we shall see that if E is sufficiently nice near ∞, all such φ are restrictions of Fourier-Stieltjes transforms, and in the second section we shall characterize the sets E, such that every bounded continuous function on E is a multiplier of $A(E)$. This gives an easy proof that any noncompact I-group contains a set E for which some ϕ satisfying (1.1) is not contained in $B(E)$.

First, however, we make explicit the connection between this problem and that of multipliers of quotients of $L_1(G)$.

If I is any closed ideal in $L_1(G)$ with hull $E \subset \Gamma$, a *multiplier* of $L_1(G)/I$ is a bounded operator T on $L_1(G)/I$ which commutes with translation.

We denote by kE the set $\{f \in L_1(G) : \hat{f} = 0, \text{ on } E\}$, which is the largest ideal with hull E (or E^- if E is not assumed closed). $\bar{f}$ will denote the coset $f + kE$. A multiplier of $L_1(G)/I$ then satisfies $T(\bar{f} * \bar{g}) = \bar{f} * T\bar{g} = T\bar{f} * \bar{g}$, for $\bar{f}$ and $\bar{g}$ in $L_1(G)/I$. In terms of Fourier transforms this says that in E, $(T\bar{f})^\wedge / \hat{f} = (T\bar{g})^\wedge / \hat{g}$ near points γ in E