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A SEMILATTICE DECOMPOSITION INTO SEMIGROUPS HAVING AT MOST ONE IDEMPOTENT

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A semigroup S is said to be *viable* if $ab = ba$ whenever ab and ba are idempotents. The main theorem of this article proves in part that S is a viable semigroup if and only if S is a semi-lattice of $\mathscr{S}$-indecomposable semigroups having at most one idempotent.

Furthermore, each semigroup appearing in the decomposition has a group ideal whenever it has an idempotent. Also included as part of the main theorem is the more elementary result that S is viable if and only if every $\mathscr{J}$-class contains at most one idempotent.

Throughout S will denote a semigroup and $E = E(S)$ the set of idemotents of S.

DEFINITION. Let $a, b \in S$. We say $a|b$ if there exist $x, y \in S$ such that $ax = ya = b$. The set-valued function $\mathfrak{M}$ on S is defined by $\mathfrak{M}(a) = \{e | e \in E, a|e\}$. The relation δ on S is defined by $a \;\delta\; b$ if $\mathfrak{M}(a) = \mathfrak{M}(b)$.

Our first goal is to show that if S is viable then δ is a congruence on S and S/δ is the semilattice described above.

LEMMA 1. *Let S be viable. If $ab = e \in E$, then $bea = e$.*

Proof. $(bea)^2 = beabea = bea$. Hence $bea \in E$. But cleary $abe = e \in E$. Hence $bea = abe = e$.

LEMMA 2. *Let S be viable. Suppose $a \in S$ and $e \in E$. Then $a|e$ if and only if $e \in S^1aS^1$.*

Proof. If $a|e$, then $e \in S^1aS^1$ by definition. Conversely assume $e = sat$ with $s, t \in S^1$. By (1), $ates = e$ and $tesa = e$. Therefore $a|e$.

THEOREM 3. *Let S be viable. Then*

(i) *δ is a congruence relation on S containing Green's relation $\mathscr{H}$.*

(ii) *S/δ is a semilattice and*

(iii) *each δ-class contains at most one idempotent and a group ideal whenever it contains an idempotent.*

Proof. (i) Clearly δ is an equivalence relation. We will show that δ is right compatible. Assume $a \;\delta\; b$. If $ac|e \in E$, then