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SEMI-ORTHOGONALITY IN RICKART RINGS

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This note initiates a study of the semi-orthogonality relation on the lattice of principal left ideals generated by idempotents of a Rickart ring. It will be seen that two left ideals in a von Neumann algebra are semi-orthogonal if and only if their unique generating projections are non-asymptotic. Connections between semi-orthogonality, dual modularity, von Neumann regularity, and algebraic equivalence will be established; those Rickart rings with a superabundance of semi-orthogonal left ideals will be characterized.

A *regular ring* is a ring A with identity in which each element $a \in A$ is *regular* in the sense that $aba = a$ for some element $b \in A$. A *Rickart ring* is a ring A with identity in which the left (and right) annihilator of each element is a principal left (right) ideal generated by an idempotent. Regular rings and Baer rings, as defined by Kaplansky [4], are special cases of Rickart rings: in particular, then, a von Neumann algebra is a Rickart ring. Rickart rings are called Baer rings in [2]. Throughout this note, A will denote a Rickart ring. $L(M)$ and $R(M)$ will denote respectively the left and right annihilators of a subset M of A. The letters e, f, g, h and k will denote idempotents and the letters E, F, G, H and K will denote the left ideals they generate.

Ordered by set inclusion, the set $L(A)$ of principal left ideals generated by idempotents forms a lattice. If E and F form a modular pair in $L(A)$, we shall write $(E, F)M$; if E and F form a dual modular pair in $L(A)$, we shall write $(E, F)M^*$. Following S. Maeda [6], we shall say that two left ideals E and F in $L(A)$ are *semi-orthogonal*, $E \sharp F$, if they are generated by orthogonal idempotents. Maeda shows that the semi-orthogonality relation $\sharp$ on $L(A)$ has these properties: (1) If $E \sharp E$, then $E = (0)$; (2) If $E \sharp F$, then $F \sharp E$; (3) If $E_1 \leqq E$ and $E \sharp F$, then $E_1 \sharp F$; (4) If $E \sharp F$ and $E \vee F \sharp G$, then $E \sharp F \vee G$; (5) If $E \leqq F$, then there is a left ideal G in $L(A)$ such that $E \vee G = F$ and $E \sharp G$.

The results herein form a portion of the author's dissertation, submitted to the Graduate School of the University of Massachusetts and directed by Professor D. J. Foulis.

2. Semi-orthogonal left ideals.

In this section, we give geometric meaning to Maeda's canonical semi-orthogonality relation in $L(A)$.