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INTEGRAL REPRESENTATION OF EXCESSIVE FUNCTIONS OF A MARKOV PROCESS

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Let X_t be a standard Markov process on a locally compact separable metric space E having a Radon reference measure. Let $\mathscr{S}$ denote the set of locally integrable excessive functions of X_t and $ex\mathscr{S}$ the set of elements lying on the extremal rays of $\mathscr{S}$. Then if $u \in ex\mathscr{S}$ is not harmonic, it is shown that there is an $x \in E$ such that $P_V u = u$ for all neighborhoods V of x where P_V is the hitting operator of V. A regularity condition is introduced which guarantees that two functions in $\mathscr{S}$ having the above property at x are proportional. A subset $\hat{E} \subset E$ and a metric topology on $\hat{E}$ are defined which allows one to represent each potential $p \in \mathscr{S}$ in the form $p(x) = \int u(x, y)\nu(dy)$ for some finite Borel measure $\nu \geqq 0$ on $\hat{E}$. Here the function $u: E \times \hat{E} \rightarrow [0, \infty]$ is measurable with respect to the product Borel field and has the property that for each $y \in \hat{E}$ the function $x \rightarrow u(x, y)$ is an extremal excessive function. In the course of this study a dual potential operator is introduced and some of its properties are investigated.

In § 2 we introduce the notation and assumptions which will be assumed to hold throughout the paper. Section 3 begins our study of $ex\mathscr{S}$ and using a result of Meyer [7] we show that to each function $u \in ex\mathscr{S}$ which is not harmonic we can associate a point $x \in E$ such that $P_V u = u$ for all open neighborhoods V of x. Here P_V is the hitting operator associated with V. We then say that u has support at x in analogy to the property introduced in axiomatic potential theory by Hervé [4]. We then discuss the axiom of proportionality, i.e., when is it true that if $u_1, u_2 \in ex\mathscr{S}$ have support at x, it follows that $u_1 = \alpha u_2$ for some $\alpha \geqq 0$. Some conditions are given which guarantee this property.

In § 4 we begin the discussion of representation of elements of $\mathscr{S}$. A uniform integrability condition on $\mathscr{S}$ is imposed and we define a suitable compact, convex set $\mathscr{K}$ in $\mathscr{S}$. Using the Choquet theorem and the characterization of $ex\mathscr{S}$ established in § 3, we define a subset $\hat{E} \subset E$ and a metric topology on $\hat{E}$ which allows us to represent each potential $p \in \mathscr{K}$ in the form $p(x) = \int u(x, y)\nu(dy)$ for some Borel measure $\nu \geqq 0$ on $\hat{E}$. Here $u: E \times \hat{E} \rightarrow [0, \infty]$ is a function measurable with respect to the product Borel field on $E \times \hat{E}$ and having the property that the function $x \rightarrow u(x, y)$ is an extremal excessive func-