PACIFIC JOURNAL OF MATHEMATICS
Vol. 39, No. 1, 1971

A DUALITY BETWEEN TRANSPOTENCE ELEMENTS AND MASSEY PRODUCTS

BYRON DRACHMAN AND DAVID KRAINES

The purpose of this note is to show that if v is an element whose suspension is nonzero, and if u is dual to v, then the transpotence $\varphi_k(v)$ is defined and nonzero if and only if the k-Massey product $\langle u\rangle^k$ is defined and nonzero.

We wish to thank Dr. Samuel Gitler for a helpful conversation on this material.

1. Preliminaries.

1.1. *The Cobar Construction*: (Adams [1]). Let C be a simply connected DGA coalgebra over K with co-associative diagonal map where K is a commutative ring with unit. The Cobar Construction $\bar{F}(C)$ is the direct sum of the n-fold tensor products of the desuspension of $\bar{C} = \text{Ker}\ (\varepsilon)$ where ε: $C \to K$ is the augmentation. Suppose C has a differential $\{d_n\colon C_n \to C_{n-1}\}$. A typical element is a linear combination of elements of the form

$$x = s^{-1}(c_1) \otimes \cdots \otimes s^{-1}(c_n) = [c_1 \mid \cdots \mid c_n]$$

where x has bidegree $(-n, m)$ and $m = \sum_{i=1}^{n}$ degree (c_i). The differential in $\bar{F}(C)$ is defined on elements of bidegree $(-1, *)$ by

$$d[c] = [-dc] + \sum_i (-1)^{\deg c_i'}[c_i' \mid c_i'']$$

where

$$\Delta(c) = c \otimes 1 + 1 \otimes c + \sum_i c_i' \otimes c_i''$$

Δ: $C \to C \otimes C$ being the diagonal mapping of C. The differential is extended to all of $\bar{F}(C)$ by the requirement that $\bar{F}(C)$ be a DGA-algebra.

If C has a differential of degree $+1$ instead of -1, we no longer ask that C be a simply connected but only connected, and the element $[c_1 \mid \cdots \mid c_n]$ is assigned bidegree (n, m).

1.2. *The Bar Construction*. Let A be a connected associative DGA algebra over K. Let ε: $A \to K$ be the augmentation. Let $\bar{A} = \ker \varepsilon$. Then the Bar Construction $\bar{B}(A)$ is the direct sum of the n-fold tensor products of the suspension of $\bar{A}$. Let

$$\{d_n\colon A_n \to A_{n-1}\}$$