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EXTENSIONS OF TOPOLOGICAL GROUPS

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In this paper, we will be concerned with topological group extensions of a polonais group A by a polonais group G. When A is abelian, we will consider two cohomology groups: $H^2(G, A)$ and $H^2_V(G, A)$. $H^2(G, A)$ is based on Borel cochains and was studied by Moore. $H^2_V(G, A)$ is obtained by identifying cochains which differ only on a first category set, an idea suggested to us by D. Wigner. We will show that each of the groups classifies the extensions and that the hypothesis that A be abelian can be eliminated.

By a polonais group, we mean a separable metrizable topological group which is complete in its two-sided uniformity. The completeness requirement is equivalent to topological completeness (see [2], Exercise $Q(d)$, p. 212). If E is a topological group having a polonais normal subgroup A such that E/A is polonais, then it is elementary to prove that E must be polonais also.

If A is abelian, then the cohomology groups are defined in terms of an a priori action of G on A. If A is non-abelian, then we will define sets (not groups) $H^2(G, A)$ without given action of G on A. For brevity, we proceed directly to the general case.

Let $\mathscr{A}$ be the group of topological automorphisms of A. We will write ${}^{\theta}a$ for the action of $\theta \in \mathscr{A}$ on $a \in A$ and I_a for the inner automorphism $b \rightarrow aba^{-1}$. Let e denote the identity of any group. Then $H^2(G, A)$ is defined by means of cocycles (σ, ρ) where:

$$\sigma: G \times G \rightarrow A, \ \rho: G \rightarrow \mathscr{A}, \tag{1}$$

σ is a Borel function on $G \times G$ and $(x, a) \rightarrow {}^{\rho(x)}a$ is a Borel function on $G \times A$.

$$\begin{aligned} \sigma(x, y)\cdot\sigma(xy, z) &= {}^{\rho(x)}\sigma(y, z)\cdot\sigma(x, yz) , \\ \rho(x)\cdot\rho(y) &= I_{\sigma(x,y)}\rho(x, y) , \\ \sigma(x, e) &= \sigma(e, y) = e, \text{ and} \\ \rho(e) &= e . \end{aligned} \tag{2}$$

(σ, ρ) and (σ', ρ') are identified in $H^2(G, A)$ if there is a Borel function $\lambda: G \rightarrow A$ such that:

$$\begin{aligned} \sigma'(x, y) &= \lambda(x)\cdot{}^{\rho(x)}\lambda(y)\cdot\sigma(x, y)\cdot\lambda(xy)^{-1} , \\ \rho'(x) &= I_{\lambda(x)}\cdot\rho(x) . \end{aligned} \tag{3}$$

$H^2_V(G, A)$ can be defined by simply stipulating that (2) and (3)