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MODULES OVER UNIVERSAL REGULAR RINGS

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To each commutive ring R there is associated a certain commutative regular ring $\hat{R}$. The ring $\hat{R}$ is in fact an R-algebra. It is shown that ${}_R\hat{R}$ is never flat, unless R is itself regular. The functor taking R to $\hat{R}$ preserves direct limits, and, in certain cases, tensor products. It is shown that if R is weakly noetherian then the global dimension of $\hat{R}$ is less than or equal to the Krull dimension of R. Necessary and sufficient conditions that $\hat{R}$ be a quotient ring of R are determined.

In this paper we study a certain commutative (von Neumann) regular ring $\hat{R}$ associated with each commutative ring R. There is a natural homomorphism $\phi: R \rightarrow \hat{R}$, characterized by the following universal property: every homomorphism from R into a regular ring factors uniquely through ϕ. The ring $\hat{R}$ has been studied briefly in [7] and [5]. In §1 we construct $\hat{R}$ and derive its basic properties, including the universal property mentioned above. The construction uses a little sheaf theory, although once a few lemmas have been proved it will rarely be necessary to recall the sheaf-theoretic construction. In fact, in §5 we give a simple description of $\hat{R}$ that is completely nontopological. In §2 we study relationships between an R-module A and the $\hat{R}$-module $A \otimes_R \hat{R}$, and in §3 we restrict our attention to weakly noetherian rings, that is, rings with maximum condition on radical ideals. It is shown that R is weakly noetherian if and only if $A \otimes_R \hat{R}$ is $\hat{R}$-projective for every finitely generated A_R. Homological considerations are taken up in §4, and it is shown that if R is weakly noetherian then the global dimension of $\hat{R}$ is less than or equal to the Krull dimension of R. In §6 we examine how the functor taking R to $\hat{R}$ behaves with respect to tensor products and direct limits. The last section is devoted to semiprime rings, and we find necessary and sufficient conditions that $\hat{R}$ be a quotient ring of R.

We make the standing assumption that all rings are commutative with unit, and all ring homomorphisms and modules are unitary. We now establish some notation to be preserved throughout the paper. Recall that Spec(R) is the set of prime ideals of R, with the Zariski topology. If S is a subset of R, we let $V(S)$ denote the (closed) subset of Spec(R) consisting of those prime ideal that contain S, and we let $D(S) = \text{Spec}(R) - V(S)$. If $x \in \text{Spec}(R)$ let k_x denote the quotient field of the domain R/x, and for each $a \in R$ let $a(x)$ be the image