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ORTHODOX SEMIGROUPS

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An orthodox semigroup is a regular semigroup in which the idempotents form a subsemigroup. The purpose of this paper is to give structure theorems for orthodox semigroups in terms of inverse semigroups and bands.

A different structure theorem for orthodox semigroups in terms of bands and inverse semigroups has already been given by Yamada in [12]; two questions posed in [12] will be answered in the negative. The present paper is the "further paper" mentioned by the author in the final paragraph of §1 [5] and in the Acknowledgement of [5].

2. Preliminaries. We use wherever possible, and usually without comment, the notations of Clifford and Preston [2]; further, for each element a in any semigroup S we define $V(a) = \{x \in S: axa = a$ and $xax = x\}$, the set of inverses of a in S.

RESULT 1 (from Theorem 4.6 [2]). *On any band B Green's relation $\mathscr{J}$ is the finest semilattice congruence and each $\mathscr{J}$-class is a rectangular band.*

Let $\phi: B \rightarrow Y$ be any homomorphism of B onto a semilattice Y snch that $\phi \circ \phi^{-1} = \mathscr{J}$. By denoting (for all $e \in B$) J_e by E_α where $e\phi = \alpha \in Y$ we obtain B as a semilattice Y of the rectangular bands $\{E_\alpha: \alpha \in Y\}$, i.e., $B = \bigcup_{\alpha \in Y} E_\alpha$ and for all $\alpha, \beta \in Y$ $E_\alpha \cap E_\beta = \square$ if $\alpha \neq \beta$, aud $E_\alpha E_\beta \subseteq E_{\alpha\beta}$. It is clear that $\{(e, \alpha) \in B \times Y: e\phi = \alpha\}$ is a subband of $B \times Y$ isomorphic to B.

RESULT 2 [9, Lemma 2.2]. Let *ρ be a congruence on a regular semigroup S. Then each ρ-class which is an idempotent of S/ρ contains an idempotent of S.*

RESULT 3 (from Theorem 13 [7]). *Let ρ be any congruence contained in $\mathscr{L}$ on any semigroup S. Then any elements a and b of S are $\mathscr{L}$-related in S if and only if $a\rho$ and $b\rho$ are $\mathscr{L}$-related in S/ρ.*

Henceforth we shall let S denote an arbitrary orthodox semigroup.

The following result is part of [3, Theorem 3]; as noted in [4] it had previously been obtained by Schein [10].