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FOURIER-STIELTJES TRANSFORMS AND WEAKLY ALMOST PERIODIC FUNCTIONALS FOR COMPACT GROUPS

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Let G be a compact group and H a closed subgroup. A function in the Fourier algebra of H can be extended to a function in the Fourier algebra of G without increase in norm and with an arbitrarily small increase in sup-norm. For G a compact Lie group, the space of Fourier-Stieltjes transforms is not dense in the space of weakly almost periodic functionals on the Fourier algebra of G.

We let G denote an infinite compact group and $\hat{G}$ its dual. We use the notation of [1, Chapters 7 and 8], [2], and [3]. Recall $A(G)$ denotes the Fourier algebra of G (an algebra of continuous functions on G), and $\mathscr{L}^\infty(\hat{G})$ denotes its dual space under the pairing $\langle f, \phi \rangle$ ($f \in A(G)$, $\phi \in \mathscr{L}^\infty(\hat{G})$). Further, note $\mathscr{L}^\infty(\hat{G})$ is identified with the C^*-algebra of bounded operators on $L^2(G)$ commuting with right translation. The module action of $A(G)$ on $\mathscr{L}^\infty(\hat{G})$ is defined by the following: for $f \in A(G)$, $\phi \in \mathscr{L}^\infty(\hat{G})$, $f \cdot \phi \in \mathscr{L}^\infty(\hat{G})$ by $\langle g, f\cdot\phi\rangle = \langle fg, \phi\rangle$, $g \in A(G)$. Also $\|f\cdot\phi\|_\infty \leqq \|f\|_A \|\phi\|_\infty$.

Let $\phi \in \mathscr{L}^\infty(\hat{G})$. We call ϕ a weakly almost periodic functional if and only if the map $f \mapsto f\cdot\phi$ from $A(G)$ to $\mathscr{L}^\infty(\hat{G})$ is a weakly compact operator. The space of all such is denoted by $W(\hat{G})$.

Let $M(G)$ denote the measure algebra of G. For $\mu \in M(G)$, the Fourier-Stieltjes transform of μ, $\mathscr{F}\mu$, is a matrix-valued function in $\mathscr{L}^\infty(\hat{G})$ defined for $\alpha \in \hat{G}$ by

$$\alpha \mapsto (\mathscr{F}\mu)_\alpha = \int_G T_\alpha(x^{-1})\ d\mu(x)\ \ (T_\alpha \in \alpha)\ .$$

We denote the closure of $\mathscr{F}M(G)$ in $\mathscr{L}^\infty(\hat{G})$ by $\mathscr{M}(\hat{G})$. In [2], we showed that $W(\hat{G})$ is a closed subspace of $\mathscr{L}^\infty(\hat{G})$, and that $\mathscr{M}(\hat{G}) \subset W(\hat{G})$ with the inclusion proper when G is a direct product of an infinite collection of nontrivial compact groups. In this paper, we show the inclusion is proper for all compact Lie groups.

We first state a standard lemma.

LEMMA 1. *Let A, B be compact subsets of a topological group G. Suppose $AB \subset U$, U an open subset of G. Then there is an open neighborhood V of the identity e of G such that $AVB \subset U$.*