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ON THE BRAUER GROUP OF Z

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Two dimensional Amitsur cohomology is computed for certain rings of quadratic algebraic integers. Together with computations of Picard groups, this yields information on the Brauer group $B(S/Z)$, for S quadratic algebraic integers, without resort to class field theory.

The classical Brauer group of central simple algebras over a field [10, X, Sec. 5] has been generalized to the Brauer group $B(R)$ of central separable R-algebras over a commutative ring R [2]. One can prove, using class field theory, that the Brauer group $B(Z)$ of the integers, is trivial. The proof is apparently well known but not in the literature, although it does appear in the dissertation of Fossum [9].

This paper is devoted to our attempt to establish this result using only an exact sequence of Chase and Rosenberg [7, p. 76]. We are able to show that if S is the integers of $Q(\sqrt{m})$ for $m = \pm 3, -1, 2$, or 5, the subgroup $B(S/Z)$ of $B(Z)$ consisting of elements split by S, vanishes.

In § 2 we develop some technical results on norms which we use in § 3 to show that the Amitsur cohomology group $H^2(S/Z, U)$ is zero whenever S is the ring of integers of a quadratic extension of the rationals. In § 4 we use a Mayer-Vietoris sequence of algebraic K-theory to show that the Picard group $\mathrm{Pic}(S \otimes_Z S) = 0$ for S the integers of $Q(\sqrt{m})$, $m = \pm 3, -1, 2$, or 5. In § 5 we use this result and an exact sequence of Chase and Rosenberg [7, p. 76] to show $B(S/Z) = 0$ for these rings.

Dobbs [8] has results relating $B(S/Z)$ to $H^2(S/Z, U)$ which together with the triviality of $B(Z)$ imply our results.

§ 2. Norms. If S is a commutative algebra over a commutative ring R, S^n denotes $S \otimes S \cdots \otimes S$, n times (here and throughout, $\otimes$ means $\otimes_R$), and $\varepsilon_i: S^n \to S^{n+1}$, $i = 0, \cdots, n$, is given by $x_0 \otimes \cdots \otimes x_{n-1} \to x_0 \otimes \cdots \otimes x_{i-1} \otimes 1 \otimes x_i \otimes \cdots \otimes x_{n-1}$. These maps satisfy $\varepsilon_i \varepsilon_j = \varepsilon_{j+1} \varepsilon_i$ for $i \leqq j$. For any ring A, $U(A)$ denotes the group of units of A. All unexplained notation and terminology is an in [7].

THEOREM 2.0. *Let M/K be a galois extension of commutative rings* [6], *with group G, and let F be an additive functor on a full subcategory $\mathscr{C}$ of the category of commutative K-algebras, and suppose M and $M \otimes_K M$ lie in $\mathscr{C}$. Then for any x in $F(M)$, $y = \sum_{g \text{ in } G} Fg(x)$*