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RESTRICTIONS OF BANACH FUNCTION SPACES

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Let X be a compact Hausdorfs space. Let $C(X)$ be the space of continuous complex-valued functions on X and A be a function algebra on X, that is a uniformly closed separating subalgebra of $C(X)$ containing the constants. If F is a closed subset of X we say that A interpolates on F if $A \mid F = C(F)$. By a positive measure μ we shall always mean a positive regular bounded Borel measure on X. Let F be a measurable subset of X. We say a subspace S of $L^p(\mu)$ interpolates on F if $S \mid F = L^p(F) = L^p(\mu_F)$, where μ_F is the restriction of μ to F. Let $H^p(\mu)$ be the closure of A in $L^p(\mu)$ where $1 \leqq p < \infty$, and let $H^\infty(\mu) = H^2(\mu) \cap L^\infty(\mu)$. One question we are concerned with here is whether interpolation of the algebra is sufficient to imply interpolation of its associated H^p-spaces. We therefore begin by obtaining necessary and sufficient conditions for a closed subspace of $L^p(\mu)$ to have closed restriction in $L^p(F)$. These condition are analogous to some obtained by Glicksberg for function algebras. Using these results we obtain theorems about interpolation of certain invariant subspaces, and then apply them to H^p-spaces. In particular we show that when A approximates in modulus and μ is any measure which is not a point-mass, $H^p(\mu)$ interpolates only on sets of measure zero. (One sees that A interpolates only on sets of measure zero, so our original question has a trivial answer for these algebras.) For uniformly closed weak-star Dirichlet algebras again the answer to our original question is affirmative. Finally we provide an example of an algebra which interpolates such that $H^\infty(\mu)$ interpolates and the $H^p(\mu)$ do not interpolate for $1 \leqq p < \infty$. I am indebted to a paper of Glicksberg for those techniques which inspired the present effort. Below we show that these techniques apply to the L^p situation and to other "similar" situations.

Glicksberg [3] has given necessary and sufficient conditions for interpolation of a closed subspace of $C(X)$. We show here that analogous theorems hold for subspaces of $L^p(X)$. Let $A \subset B$ be Banach spaces. $A^\perp$ will denote all bounded linear functions functionals on B which annihilate A.

THEOREM 1.1. *Let A, A_1, B all be Banach spaces with $A \subset A_1$ and $R\colon A_1 \to B$ a nonzero bounded linear transformation. Then $R(A)$ is closed in B if and only if $\exists c \ni : \| h - R(A)^\perp \| \leqq c \| h^* - A^\perp \|$ $\forall h \in B^*$, where $h^* = R^*h$. It follows that $c \geqq 1/\| R \|$.*