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A MAP-THEORETIC APPROACH TO DAVENPORT-SCHINZEL SEQUENCES

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An (n, d) Davenport-Schinzel Sequence (more briefly, a DS sequence) is a sequence of symbols selected from $1, 2, \cdots, n$, with the properties that (1) no two adjacent symbols are identical, (2) no subsequence of the form abab... has length greater than d, (3) no symbol can be added to the end of the sequence, without violating (1) or (2). It is shown that the set of $(n, 3)$ DS sequences is in one-to-one correspondence with the set of rooted planar maps on n vertices in which every edge of the map is incident with the root face. The number of such sequences and the number of such sequences of longest possible length $2n-1$ is explicitly determined.

As an illustration of DS sequences, take $n = 4$, $d = 3$. Then it is simple to enumerate all DS sequences in normal form (the symbols occur in increasing order). The results follow, and we see that there are 11 (4, 3) DS sequences.

1213141
121341
1213431
123141
1232141
123241
1232421
123421
123431
1234321
123431.

It is obvious from the definition that a normal $(n, 3)$ DS sequence must begin and end with 1.

The concept of a DS sequence was introduced in [1], and various results were obtained. For subsequent developments, one may consult [2], [3], and [4].

Among all DS sequences for fixed n and d, some will have greatest lengths. We define the number $N_d(n)$ to be this greatest length. For example, in the preceding example, $N_3(4) = 7$, and there are 5 sequences of this greatest length.