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MAXIMUM MODULUS THEOREMS FOR ALGEBRAS OF OPERATOR VALUED FUNCTIONS

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Let F be a family of functions on subsets of a real Euclidean space E into a commutative subalgebra with identity T_0 of the algebra T of linear transformations of E into itself. If a suitable integration condition, motivated by Morera's theorem in complex function theory is placed on the elements of F, F becomes an algebra of "integrable" functions which can be realized as the derivatives of transformations of E into itself. It is asked what properties of the algebra of complex analytic functions from the complex plane K into K are satisfied by such algebras F. Simple examples show that analyticity and even differentiability are lost. However various forms of the maximum modulus theorem are still satisfied. Three such theorems are presented here:

(A) If commutivity of T_0 is replaced by the requirement that the elements of T_0 are "orientation preserving" then the elements of F are maximized on the boundary of a sphere.

(B) There exists $N > 0$, such that for all $f \in F$,

$$\bar{U} = \{t \in E;\ \|t\| \leqq 1\} \subseteq \textbf{domain } f,\ x \in \bar{U}\ ,$$

implies

$$\|f(x)\| \leqq N \sup\{\|f(t)\|;\ \|t\| = 1\}\ .$$

(C) For all $f \in F$, $\bar{U} \subseteq$ domain f, $x \in \bar{U}$, implies

$$\|f(x)\|_s \leqq \sup\{\|f(t)\|_s;\ \|t\| = 1\}\ ,$$

where for $A \in T_0$, $\|A\|_s$ is the spectral norm of A.

1. Introduction. The theory of complex valued integrable functions was developed by Heffter [1], Macintyre and Wilbur [6], and this author [4]. The generalization to the operator valued case was introduced in [5].

The first two results of this paper employ degree theoretic methods from algebraic topology as developed in [3] to obtain similar results. The methods of [3] represent a generalization to higher dimensional speces of methods of G. T. Whyburn [8] in the plane, employed by him to handle complex analytic functions. The third result follows from a construction of the spectral norm of an operator employing irreducible subspaces invariant under a family of operators.

Let K denote the complex plane. For $a \in K$, set $A_a(z) = az$ for