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ASYMPTOTICS FOR A CLASS OF WEIGHTED EIGENVALUE PROBLEMS

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Abstract: This paper deals with the asymptotic behavior at infinity of the solutions to $\ell(y) = \lambda wy$ on $[a, \infty)$ where ℓ is an nth order ordinary linear differential operator, λ is a nonzero complex number and w is a suitably chosen positive valued continuous functions. As an application the deficiency indices of certain symmetric differential operators in Hilbert space are computed.

1. **Preliminaries.** Throughout the first three sections ℓ will denote an operator of the form,

$$\ell(y) = y^{(n)} + \sum_{k=2}^{n} p_k y^{(n-k)} \quad \text{on } [a, \infty) , \tag{1.1}$$

where each of $p_2, \cdots, p_n$ is a continuous complex valued function on $[a, \infty)$. In view of the transformation indicated on p. 309 of [2] it results in no great loss of generality to take the coefficient of $y^{(n-1)}$ to be zero, and in order to simplify the exposition we shall do this. We shall be concerned with the behavior at infinity of the solutions to

$$\ell(y) = \lambda wy \quad \text{on } [a, \infty) \tag{1.2}$$

where λ is a nonzero complex number and w is an appropriate weight (i.e., positive valued continuous function). For a given ℓ we shall consider the weights w indicated by the following definition. $\mathscr{L}(a, \infty)$ denotes the Banach space of all complex valued measurable functions which are absolutely Lebesgue integrable on $[a, \infty)$.

DEFINITION. If ℓ is as in 1.1 the statement that w is an ℓ-admissible weight means that

(1) w is differentiable, strictly increasing, and unbounded on $[a, \infty)$;

(2) each of $[w'/w^{1+1/n}]'$ and $[(w'/w)^2(1/w^{1/n})]$ is continuous on $[a, \infty)$ and is in $\mathscr{L}(a, \infty)$; and

(3) $p_j/w^{(j-1)/n} \in \mathscr{L}(a, \infty)$ for $j = 2, 3, \cdots, n$.

For example if $\ell(y)(t) = y''(t) \pm t^\alpha y(t)$ for $t \geqq 1$ and $w(t) = t^\beta$ then w will be an ℓ-admissible weight if and only if $\beta > 0$ and $\beta > 2(\alpha + 1)$.

We shall demonstrate that when w is an ℓ-admissible weight the solutions of 1.2 have a particularly simple asymptotic behavior and