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BAXTER'S THEOREM AND VARBERG'S CONJECTURE

BENNETT EISENBERG

It is shown that Baxter's strong law for Gaussian processes provides necessary and sufficient conditions for equivalence for a wide class of Gaussian processes.

Let X be a Gaussian process with zero mean and covariance $R(s, t) = E(X(s)X(t))$. Baxter [1] proved that if R is continuous for $0 \leqq s, t \leqq 1$ with uniformly bounded second derivatives for $s \neq t$, then

$$D^+(t) = \lim_{s \downarrow t} \frac{R(s, t) - R(t, t)}{s - t}$$

and

$$D^-(t) = \lim_{s \uparrow t} \frac{R(t, t) - R(s, t)}{t - s}$$

exist and are continuous and with probability one

$$\lim_{n \to \infty} \Sigma \left[x\left(\frac{k}{2^n}\right) - x\left(\frac{k-1}{2^n}\right)\right]^2 = \int_0^1 D^-(t) - D^+(t)\, dt\,.$$

It follows that if Y is another Gaussian process with mean zero and covariance S continuous with bounded second derivatives for $s \neq t$ and if there exists a t with

$$D_R^-(t) - D_R^+(t) \neq D_S^-(t) - D_S^+(t)$$

then the measures μ_x and μ_y for the processes X are singular.

In the case where R and S are triangular covariances Varberg [8] has obtained a converse to this result. Varberg's Theorem:

$$\text{Let } R_i(s, t) = \begin{cases} u_i(s)v_i(t) & s \leqq t \\ v_i(s)u_i(t) & s \geqq t \end{cases},$$

where $i = 1$ or 2.
Assume furthermore

(A) $u_i(0) = 0$
(B) $v_i(t) > 0$ on [0, T]
(C) u_i'' and v_i'' exist and are continuous on $[0, T]$
(D) $v_i(t)u_i'(t) - u_i(t)v_i'(t) > 0$ on $[0, T]$

then μ_x and μ_y are mutually absolutely continuous if and only if

$$v_1(t)u_1'(t) - u_1(t)v_1'(t) = v_2(t)u_2'(t) - u_2(t)v_2'(t)$$