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ESTABLISHING ISOMORPHISM BETWEEN TAME PRIME KNOTS IN E^3

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The "formula" of a polygonal knot in E^3 is defined by appropriate labeling of the crossings in the regular projection of the knot. Admissible transformations of such formulas are defined (for example, cancellation of the consecutive symbols x and x^{-1}), and prime formulas are defined. It is shown that if two knots have formulas which are equivalent by applications of admissible transformations, and one of the formulas is prime, then the knots are equivalently embedded in E^3.

Since each tame knot type includes a finite polygon, we restrict our attention to polygonal knots in E^3. Such a knot is the image of a one-to-one continuous mapping g of $[0, 1)$ into E^3 such that (1) $g(t)$ approaches $g(0)$ as t approaches 1, and (2) the image of g is the union of a finite number of straight line intervals. We may of course restrict our attention to such knots $K = \mathrm{Im}(g)$ as lie in general position in E^3; that is, π (defined by $\pi(x, y, z) = (x, y, 0)$) is one-to-one on K except at a finite number of points, called the double points of K, where π is precisely two-to-one, and no vertex of K is a double point.

Let $x_1, x_2, \cdots, x_n$ be the points of $[0, 1)$ mapped two-to-one by $f = \pi g$, arranged in their natural order. The formula of the knot K is then

$$f(x_1)^{e(1)} f(x_2)^{e(2)} \cdots f(x_n)^{e(n)}$$

where $e(i)$ is 1 or -1 according to the following rule: If $f(x_i) = f(x_j)$ and the z-coordinate of $g(x_i)$ exceeds that of $g(x_j)$, then $e(i) = 1$ and $e(j) = -1$. In practice we suppress the positive superscripts. For example, the formula of the trefoil knot drawn in the ordinary way can be written $ab^{-1}ca^{-1}bc^{-1}$, where a, b, and c are the three crossings in the plane projection of the trefoil. If there are no double points, the knot has empty formula denoted by 1.

Let a knot formula F be given. By an admissible operation on F is meant the application to F of one of the following ten transformations.

(1) Reversal of the order of symbols of F.

(2) Coding; that is, consistent substitution of different symbols for the symbols of F, while preserving superscripts.

(3) Negation of all superscripts in F.

(4) Cyclic permutation of the symbols of F, as for example re-