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COVERING RELATIONS AMONG LATTICE VARIETIES

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0. Introduction. It is shown in this paper that the equational class generated by the family of all projective planes can be characterized by a finite set of lattice identities. The methods developed here may provide a framework to attack similar problems and a useful medium for studying modular lattices in general.

By a variety, or equational class, of lattices we mean the class of all lattices satisfying a given set of lattice identities. A lattice variety is finitely based if it can be defined by a finite set of identities. Let Λ be the lattice of all lattice varieties. A systematic study of the lattice Λ dates back seven or eight years ago. Most recent results in this field, including ours here, are stimulated by an important discovery of Bjarni Jónsson in [7], Corollary 3.2. (See Baker [1], [2], Grätzer [4], Hong [5], Jónsson [7], [8], McKenzie [9], [10], Wille [11].) Our study here continues the works of Grätzer in [4] and of Jónsson in [8], where the latter completed an unfinished result of the former and in particular proved that the variety generated by all projective lines is finitely based.

The rest of the paper is divided into four sections. In §1 we state our main theorem and its applications but postpone the proofs until §4. In §2 we discuss the main methods employed here: the method of strong covering, and the notions of normality and strong normality of sequences of transposes. In case the family of all varieties that strongly covers a given variety is finite, then the variety is finitely based. The notions of normality and strong normality, due to Grätzer and Jónsson respectively, are developed rather completely in Theorem 3.1. We hope that this theorem will have some applications elsewhere. Section 4 gives details of the proof of the main lemma stated in Section 1.

In the sequel, almost every theorem and lemma has its dual, even though we rarely make explicit mention of this fact. Also, the notation L denotes a fixed modular lattice.

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1. The main theorem and its applications. For any family $\boldsymbol{K}$ of lattices, let $\boldsymbol{S}(\boldsymbol{K})$, $\boldsymbol{H}(\boldsymbol{K})$, $\boldsymbol{P}_u(\boldsymbol{K})$ denote respectively the families of sublattices, of homomorphic images and of ultraproducts of members