PACIFIC JOURNAL OF MATHEMATICS
Vol. 40, No. 3, 1972

# TWO BRIDGE KNOTS ARE ALTERNATING KNOTS

R. E. GOODRICK

**H. Schubert introduced a numerical knot invariant called the bridge number of a knot. In particular, he classified the two-bridge knots and proved that they were prime knots. Later, Murasugi showed that if $K$ is an alternating knot then the matrix of $K$ is alternating. The latter is true of two-bridge knots. The purpose of the following is to give a somewhat unusual geometric presentation of two-bridge knots from which it will be seen that they are alternating knots.**

By a knot we will mean a polygonal simple closed curve in $E^3$. Let $C$ denote the unit circle in the $xy$ plane and $f$ a homeomorphism from $C$ to a knot $K$. We will assume that $K$ is in a regular position with respect to a projection into the $y = 0$ plane [1] and that those points of $K$ which do not have unique images will be the crossing points of $K$. Let $f^{-1}(a_1), f^{-1}(a_2), \cdots, f^{-1}(a_{2n})$ be the points of $C$ ordered clockwise where $a_1$ are the crossing points of $K$. If $K$ has a presentation with an associated $f$ such that $a_i$ is an overcrossing point if and only if $i$ is odd, then $K$ is said to be an alternating knot. By a two-bridge knot we mean a nontrivial knot in $E^3$ which can be represented by two linear segments through a convex cell and two arcs on the boundary of the cell.

THEOREM 1. *If $K$ is a two-bridge knot, then $K$ is an alternating knot.*

*Proof.* We will start with $K$ in a two-bridge representation (Fig. 1a) and apply several space homeomorphisms to $E^3$, so that the resulting representation of $K$ is described by an arc 'monotonely' approaching the center of the cube and four linear segments (Fig. 1b). The proof

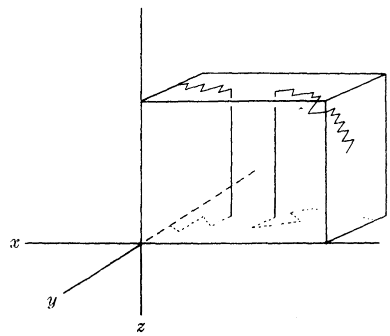


Figure 1a.

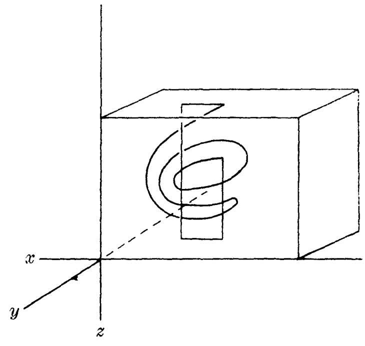


Figure 1b