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COMPLETELY ADEQUATE NEIGHBORHOOD SYSTEMS AND METRIZATION

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In this paper, the notion of a completely adequate neighborhood system for a topological space is defined and used to obtain characterizations of discreteness and second countability. Certain conditions on the completely adequate neighborhood system are given which yield collection wise normality and paracompactness. The notion of a standardized topological space is introduced (the class of standardized spaces includes, among others, the separable spaces and the developable spaces) and the main theorem gives necessary and sufficient conditions for the metrizability of standardized spaces in terms of completely adequate neighborhood systems.

0. Introduction and preliminaries.

A great deal of work has been done in the area of metrization criteria for T_1 topological spaces. Roughly speaking, these criteria generally fall into two broad categories which might be characterized as "indexed neighborhood" criteria and "covering" criteria. In this paper, an "unindexed neighborhood" criterion is developed, that is, a metrization criterion in which the set algebraic conditions which the neighborhoods of points must satisfy is separated from the indexing requirement.

Specifically, the set algebraic requirement is that the space have a linearly ordered, completely adequate neighborhood system (defined in §1), and the indexing requirement is that the space have a standardization (defined in §2).

In what follows, all spaces are assumed to be T_1. A "neighborhood system" will always mean a mapping, $\mathscr{U}$, which assigns to each $x \in X$ a neighborhood basis at x, denoted by $\mathscr{U}(x)$. For any subset, A, the closure of A will be denoted by $\mathrm{Cl}[A]$, and its interior by $\mathrm{Int}[A]$. Otherwise, notation and terminology will follow that of Kelley [3].

1. Completely adequate neighborhood systems.

DEFINITION 1.1. A completely adequate neighborhood system (henceforth abbreviated to C. A. system) is a map, $\mathscr{V}$, which assigns to each $x \in X$ a neighborhood basis, $\mathscr{V}(x)$, at x, satisfying:

Given $x \in X$ and 0 open, with $x \in 0$, there exists an open set $N[x; 0]$, with $x \in N[x; 0] \subset 0$, and such that each $y \in N[x; 0]$ has a neighborhood $V(y) \in \mathscr{V}(y)$ with the property that $N[x; 0] \subset V(y) \subset 0$.