GENERALIZED RAMSEY THEORY FOR GRAPHS, III.
SMALL OFF-DIAGONAL NUMBERS

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The classical Ramsey theory for graphs studies the Ramsey numbers $r(m, n)$. This is the smallest p such that every 2-coloring of the lines of the complete graph K_p contains a green K_m or a red K_n. In the preceding papers in this series, we developed the theory and calculation of the diagonal numbers $r(F)$ for a graph F with no isolated points, as the smallest p for which every 2-coloring of K_p contains a monochromatic F. Here we introduce the off-diagonal numbers: $r(F_1, F_2)$ with $F_1 \neq F_2$ is the minimum p such that every 2-coloring of K_p contains a green F_1 or a red F_2. With the help of a general lower bound, the exact values of $r(F_1, F_2)$ are determined for all graphs F_i with less than five points having no isolates.

1. **Introduction.** The small ($p \leq 4$ points) graphs F_i having no isolated points are shown in Figure 1, together with their symbolic names, following the notation for operations on graphs in the book [3, p. 21]. In fact, we follow the terminology and notation of this book throughout.
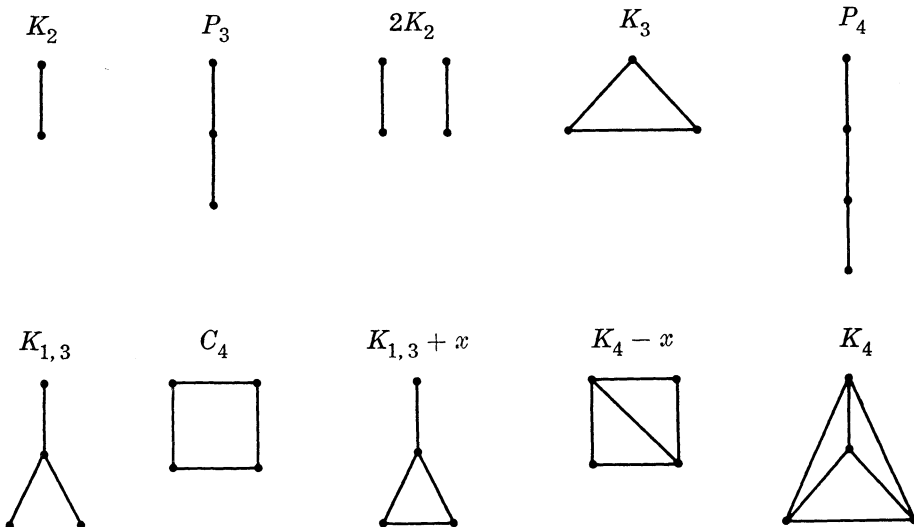

FIGURE 1

In [1, 2], we defined the number $r(F)$ as the minimum p for which every 2-coloring (of the lines) of K_p contains a monochromatic subgraph F. The number $r(F_1, F_2)$ is the corresponding smallest p