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HOMOMORPHISMS OF BANACH ALGEBRAS WITH MINIMAL IDEALS

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Let A be a semi-simple Banach algebra with socle F, and let ν be a homomorphism of A into a Banach algebra. It is shown that if I is a minimal one-sided ideal of A, then the restriction of ν to I is continuous. This is then used to deduce continuity properties of the restriction of ν to F. In particular, if F has a bounded left or right approximate identity, then ν is continuous on F.

In [1] and [2] we deduced continuity properties of $\nu|F$ in case A was a semi-simple annihilator Banach algebra. In this paper we obtain essentially the same results, but without the hypothesis that A be an annihilator algebra.

We first show that the restriction of ν to any minimal one-sided ideal is continuous. The proof is almost purely algebraic. We then show that there exists a constant K such that

$$\| \nu (xy) \| \leqq K \| x \| \| y \| , \quad x \in F , \quad y \in \bar{F} .$$

As a corollary we obtain that $\nu|F$ is continuous if F has a bounded left or right approximate identity.

1. Preliminaries. Throughout this section we assume that A is a complex semi-simple Banach algebra. The *socle*, F, is defined to be the sum of the minimal right ideals. An idempotent e is called *minimal* if eA is a minimal right ideal. We use without reference the basic facts about the socle of a Banach algebra (see e.g. [7, pp. 45-47]).

The following two lemmas, together with the "Main Boundedness Theorem" of Bade and Curtis ([3, Thm. 2.1], [2, Thm. 4.1]) are the basic ingredients in the proofs that follow. The first lemma is due essentially to Barnes.

LEMMA 1.1. *Let* $\{x_1, \cdots, x_n\} \subset F$. *Then there exist idempotents* e *and* f *in* F *such that* $\{x_1, \cdots, x_n\} \subset eAf$ *and* eAf *is finite-dimensional.*

Proof. By hypothesis, there exist minimal right ideals, $I_1, \cdots, I_m$, whose sum contains $\{x_1, \cdots, x_n\}$. By [4, Thm. 2.2], there exists an idempotent $e \in F$ such that $eA = I_1 + \cdots + I_m$. Thus $x_k \in eA$, $1 \leqq k \leqq n$.