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REGULAR SEMIGROUPS WHICH ARE EXTENSIONS OF GROUPS

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A semigroup V is an (ideal) extension of a semigroup T by a semigroup S with zero if T is an ideal of V and S is isomorphic to the Rees quotient V/T. Considered here are those semigroups which can be constructed as an extension of a group by a 0-categorical regular semigroup. The multiplication in such a semigroup is determined, along with an abstract characterization of the semigroup.

Let G be a group and S a 0-categorical regular semigroup. The problem of finding all extensions of G by S is essentially that of determining the associative multiplications on the set $V = G \cup (S\backslash 0)$ which make G an ideal of V. Such multiplications are characterized here completely in so far as semigroups are concerned. This description is made possible by a new use of the minimal primitive congruence on S as defined by T. E. Hall in [3].

Finally, having made such extensions, we give a characterization of those semigroups which can be constructed in this manner, that is, as an extension of a group by a 0-categorical regular semigroup.

1. Preliminary remarks. For a semigroup S with zero, let S^* denote $S\backslash 0$, and E_S be the set of idempotents of S. Letting T be any semigroup, a function $\theta: S^* \rightarrow T$ satisfying the condition

$$(a\theta)(b\theta) = (ab)\theta \quad \text{if } ab \neq 0 \text{ in } S$$

is called a *partial homomorphism of S into T.*

By Theorem 4.19 of [2], every extension of a group by an arbitrary semigroup S with zero is completely determined by a partial homomorphism of S into the group. It is our task here to characterize all such functions in the case that S is a 0-categorical regular semigroup.

A subset A of a semigroup S is called *categorical* if for a, b, c in S, $abc \in A$ implies that $ab \in A$ or $bc \in A$. If S has a zero and $\{0\}$ is a categorical subset of S, then S is called *0-categorical* or *categorical at* 0.

Examples of 0-categorical semigroups include Rees matrix semigroups, primitive regular semigroups, ω-regular semigroups (see [1]),