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A GENERALIZATION OF THE PRIME RADICAL IN NONASSOCIATIVE RINGS

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In [5] Tsai defined the Brown-McCoy prime radical for Jordan rings in terms of the quadratic operation and proved basic results for the radical. In this paper we give a definition of the prime radical for arbitrary nonassociative rings in terms of a $*$-operation defined on the family of ideals and of a function f of the ring into the family of ideals in the ring. The prime radical for Jordan or standard rings is obtained by a particular choice of the $*$-operation and the function f. We also extend the results for the Jordan case to weakly W-admissible rings which include the generalized standard rings and therefore alternative and standard rings as well as Jordan rings.

1. Let K be any nonassociative ring and let $\mathscr{I}(K)$ denote the family of ideals of K.

DEFINITION 1. We define a $*$-operation as a mapping of $\mathscr{I}(K) \times \mathscr{I}(K)$ into the family of additive subgroups of K such that

($*$1) for A, B, C, and D in $\mathscr{I}(K)$ if $A \subseteqq C$ and $B \subseteqq D$, then $A*B \subseteqq C*D$,

($*$2) $(0)*A = B*(0) = (0)$ for all A, B in $\mathscr{I}(K)$,

($*$3) $\overline{A*B} = \bar{A}*\bar{B}$ for any homomorphic images $\bar{A}$ and $\bar{B}$ of A and B in $\mathscr{I}(K)$.

If K is a Jordan ring, let $U_x \equiv 2R_x^2 - R_{x^2}$ be the quadratic operation and AU_B be the additive subgroup of K generated by xU_y, $x \in A$ and $y \in B$. Then the U-operation satisfies the conditions above. If the characteristic is not 2, it is shown in [5] that $AU_A = AA^2$ and is an ideal of K for A in $\mathscr{I}(K)$.

For any ring K and A, B in $\mathscr{I}(K)$, if we define $A*B$ as the additive subgroup $AB^2 + B^2A + (AB)B + (BA)B$, then $A*B$ also satisfies the conditions in Definition 1. In case K is a standard ring, it is shown in [6] that $A*B$ is an ideal of K for A, B in $\mathscr{I}(K)$. If K is commutative or anticommutative, then $A*B = AB^2 + (AB)B$. In particular, if K is a Lie ring, $A*B$ is an ideal of K. Since A^2 is not in general an ideal of K for A in $\mathscr{I}(K)$, but there are considerably broad classes of nonassociative rings in which $A^3 \equiv AA^2 + A^2A$ is an ideal of K for every ideal A, this example will be particularly interesting.

We recall that a noncommutative Jordan ring K is one satisfying