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A CHARACTERIZATION OF GENERAL Z.P.I.-RINGS II

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A commutative ring R is a general Z.P.I.-ring if each ideal of R can be represented as a finite product of prime ideals. If R is not a general Z.P.I.-ring, it is still possible that each principal ideal of R can be represented as a finite product of prime ideals. In this paper, it is shown that if R is a commutative ring in which each ideal generated by two elements can be written as a finite product of prime ideals, then R must be a general Z.P.I.-ring.

Let R be a commutative ring. R is a *general Z.P.I.-ring* if each ideal of R can be represented as a finite product of prime ideals. In a previous paper, we proved that R is a general Z.P.I.-ring if each finitely-generated ideal of R can be represented as a finite product of prime ideals [4; Theorem 2.3]. If each ideal of R generated by n or fewer elements can be represented as a finite product of prime ideals, then we define R to be a *$\pi(n)$-ring*. Mori completely characterized the structure of $\pi(1)$-rings in a series of four papers [5, 6, 7, 8]. Using his characterization, it is not difficult to construct a $\pi(1)$-ring that is not a $\pi(n)$-ring for any $n > 1$. For this reason it is surprising that the main result of this paper is the following theorem.

THEOREM. *Let R be a commutative ring. Then the following conditions are equivalent:*

(a) *R is a general Z.P.I.-ring;*

(b) *for $n \geqq 2$, R is a $\pi(n)$-ring;*

(c) *R is a $\pi(2)$-ring.*

Throughout this paper, R denotes a commutative ring and n denotes an arbitrary positive integer.

2. **$\pi(n)$-rings without zero-divisors.** If D is an integral domain, we call a prime ideal P of D *minimal* if P is of height one. An integral domain D with identity is a *Krull domain* if there is a set of rank one discrete valuation rings $\{V_\alpha\}$ such that $D = \bigcap_\alpha V_\alpha$ and such that each nonzero element of D is a non-unit in only finitely many of the V_α.

EXAMPLE 2.1. An integral domain D with identity is a $\pi(1)$-ring if and only if D is a Krull domain in which each minimal prime ideal