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ALGEBRAS OF ANALYTIC FUNCTIONS IN THE PLANE

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Let X be a compact subset of the complex plane and let A be an algebra of functions analytic near X which contains the polynomials and is complete in its natural topology. This paper is concerned with determining the spectrum of A and describing A in terms of its spectrum. It is shown that the spectrum of A is formed from the disjoint union of certain compact subsets of C (suitably topologized) by making certain identifications. A is closed under differentiation exactly when no identifications need be performed, and then A admits a simple, complete description. In particular, if X is connected, then the completion of A is merely the restriction to X of the algebra of all functions analytic near the union of X with some of the bounded components of $C - X$.

Our principal tool in these investigations is the theory of analytic structure in the spectrum of a function algebra developed by Bishop in [2] and extended by Bjork in [4, 5]. We view the algebra A as the inductive limit of function algebras and induce analytic structure in the spectrum of A. When A is closed under differentiation, topological considerations lead quickly to the desired results. In the general case, we pass to the smallest algebra B containing A which is closed under differentiation. By introducing differentiation in the spectrum of A, we show that every continuous complex-valued homomorphism of A may be extended to B. It follows that the spectrum of A is obtained from the spectrum of B by making certain identifications. When no identifications need be performed, $A = B$.

2. **Preliminaries.** If U is an open set, we let $\mathscr{O}(U)$ denote the algebra of functions analytic on U, endowed with the topology of uniform convergence on compact sets. If V is an open subset of U, we let $r_{UV}: \mathscr{O}(U) \to \mathscr{O}(V)$ be the restriction. If X is a compact set, $\mathscr{O}(X)$ denotes the algebra of functions on X which have analytic extensions to a neighborhood of X. We view $\mathscr{O}(X)$ as the inductive limit (in the sense of functions) of the system $\{\mathscr{O}(U); r_{UV}\}$ and equip $\mathscr{O}(X)$ with the inductive limit topology; i.e., the finest topology rendering the restriction maps $r_U: \mathscr{O}(U) \to \mathscr{O}(X)$ continuous.

If A is a subalgebra of $\mathscr{O}(X)$ and U is an open set containing X, we let $A(U) = \{f \in \mathscr{O}(X): f \mid X \in A\}$. Similarly, if K is a compact set containing X, we let $A(K) = \{f \in \mathscr{O}(K): f \mid X \in A\}$. For compact sets K, L with $K \supset L$, we let $r_{KL}: \mathscr{O} \to \mathscr{O}(L)$ be the restriction. Then it is easy to see that: