PACIFIC JOURNAL OF MATHEMATICS
Vol. 42, No. 3, 1972

USING BRICK PARTITIONINGS TO ESTABLISH CONDITIONS WHICH INSURE THAT A PEANO CONTINUUM IS A 2-CELL, A 2-SPHERE OR AN ANNULUS

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Using brick patitioning, sufficient conditions are established for a subset of a Peano space to be locally euclidean. If M is a Peano space with no local cut points and S is a subcontinuum of M, has no local cut points, is the closure of a domain in M, has connected complement and contains a point x such that every simple closed curve in S not passing through x separates M, then S is a closed 2-cell, a 2-sphere or an annulus.

Three corollaries to the main theorem are started here. If M is a Peano space with no local cut points and for each point $x \in M$, there is a neighborhood U of x such that every simple closed curve in $U - x$ separates M, then M is a 2-manifold. If M is a Peano space with no local cut points and, for some $m \geqq 1$, every disjoint union of m simple closed curves separates M, then M is a 2-manifold. If M is a Peano continuum with no local cut points having a collection C of $m(m \geqq 0)$ simple closed curves such that any simple closed curve in M belongs to C if and only if it does not separate M, then (1) M is a 2-manifold with boundary $\cup C$ and (2) M is a subspace of a 2-sphere.

The main theorem and first corollary are generalizations of theorems proved by Gail Young in [6].

The proof of the main theorem uses brick partitionings, the Kline sphere characterization [3], and the construction used in the proof of the Kline theorem to show that a certain set satisfies the conditions of Zippin's characterization of a closed 2-cell [5; page 92, Theorem 5.2]. R. H. Bing developed the concept of partitioning in [2] to solve the convex metric problem. Bing first proved the Kline theorem in [1]. He proved the Kline theorem again in [3] by using brick partitionings. When we speak of the Kline theorem in this paper, we shall speak of the form of the theorem in [3]. Thus the Kline theorem and the main theorem are closely related. The relationship is apparently best observed by the use of brick partitionings.

Several other corollaries to the main theorem are presented.

1. Preliminaries.

For definitions of standard point set terms, the reader is referred to [5], while for terms concerning partitioning the reader is referred to [3].

We begin by making precise certain terms which are not universal.