PACIFIC JOURNAL OF MATHEMATICS
Vol. 42, No. 3, 1972

A CLASS OF BILATERAL GENERATING FUNCTIONS FOR CERTAIN CLASSICAL POLYNOMIALS

J. P. SINGHAL AND H. M. SRIVASTAVA

In this paper the authors first prove a theorem on bilateral generating relations for a certain sequence of functions. It is then shown how the main result can be applied to derive a large variety of bilateral generating functions for the Bessel, Jacobi, Hermite, Laguerre and ultraspherical polynomials, as well as for their various generalizations. Some recent results given by W. A. Al-Salam [1], S. K. Chatterjea [2], M. K. Das [3], S. Saran [6] and the present authors [9] are thus observed to follow fairly easily as special cases of the theorem proved in this paper.

Let the sequence of functions $\{S_n(x) \mid n = 0, 1, 2, \cdots\}$ be generated by

$$\sum_{n=0}^{\infty} A_{m,n} S_{m+n}(x) t^n = \frac{f(x, t)}{[g(x, t)]^m} S_m(h(x, t)) , \tag{1}$$

where m is a nonnegative integer, the $A_{m,n}$ are arbitrary constants, and f, g, h are arbitrary functions of x and t.

In the present paper we first prove the following

THEOREM. *For the* $S_n(x)$ *generated by* (1), *let*

$$F[x, t] = \sum_{n=0}^{\infty} a_n S_n(x) t^n , \tag{2}$$

where the $a_n \neq 0$ *are arbitrary constants.*

Then

$$\begin{aligned} &f(x, t) F[h(x, t), yt/g(x, t)] \\ &\quad = \sum_{n=0}^{\infty} S_n(x) \sigma_n(y) t^n , \end{aligned} \tag{3}$$

where $\sigma_n(y)$ *is a polynomial of degree* n *in* y *defined by*

$$\sigma_n(y) = \sum_{k=0}^{n} a_k A_{k,n-k} y^k . \tag{4}$$

We also show how this theorem can be applied to derive a large number of bilateral generating functions for those classical polynomial systems that satisfy a relationship like (1). In particular, we discuss the cases of the Bessel, Jacobi, Hermite, Laguerre and ultraspherical polynomials.