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# COMPOSITION CONSTRUCTIONS ON DIFFEOMORPHISMS OF $S^p \times S^q$

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**It is shown that a map from $S^{p+r}$ to $S^p$ in the image of the $J$-homomorphism sends particular types of diffeomorphisms of $S^p \times S^q$ into diffeomorphisms of $S^{p+r} \times S^q$. This is applied to the problem of determining the diffeomorphism type of an exotic $(p+q+r+1)$-sphere obtained by attaching $D^{p+r+1} \times S^q$ to $S^{p+r} \times D^{q+1}$ via a diffeomorphism of $S^{p+r} \times S^q$.**

The first section of this paper deals with the generalities of the construction. In the next two sections we exploit these ideas in a study of the plumbing pairing (compare [4]), and we show that the pairing vanishes in an infinite class of cases (3.2) and some selected low dimensions of other types (3.4). The last section of this paper uses the basic construction to investigate the existence of smooth semifree circle actions on homotopy nine spheres [5]. In particular, it is shown that if $\Sigma^9$ does not bound a spin manifold, then it has no semifree circle action with 5-dimensional fixed point set. See [23] for further nonexistence theorems concerning semifree circle actions on homotopy spheres and some geometric applications.

## 1. Constructions for compositions.

Let $\alpha \in \pi_{p+r}(S^p)$ and $\beta \in \pi_p(SO_{q+1})$ be given. Then it is well known that $\beta$ induces a diffeomorphism of $S^p \times S^q$ and the composition $\beta \cdot \alpha \in \pi_{p+r}(SO_{q+1})$ induces a diffeomorphism of $S^{p+r} \times S^q$. If $\alpha$ is in the image of $J: \pi_r(SO_p) \to \pi_{p+r}(S^p)$ we shall give a geometric procedure for passing from the diffeomorphism induced by $\beta$ to that induced by $\beta \cdot \alpha$. In all our applications $\alpha$ will be the nontrivial homotopy class in $\pi_{p+1}(S^p)$.

PROPOSITION 1.1. *Let $X$ be an $H$-space, let $\gamma \in \pi_r(SO_p)$, and let $\beta \in \pi_p(X)$, where $p \geqq 2$. If $h$ is the diffeomorphism of $S^r \times S^p$ induced by $\gamma$, the map $\pi: S^r \times S^p \to S^p$ is projection, and $q: S^r \times S^p \to S^{p+r}$ is the collapsing map, then the following formula holds*:

$$h^* \pi^* \beta = \pi^* \beta \cdot q^*(\beta \cdot J(\gamma)) .$$

The dot represents multiplication in $[S^p \times S^r, X]$. The above result was proved for $X$ a double loop space in [21, Appendix].

*Proof.* Without loss of generality the diffeomorphism $h$ maps $* \times D^{r+1}$ to itself by the identity ($*$ is the basepoint). Thus if