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# COMMUTANTS OF SOME HAUSDORFF MATRICES

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**Let $B(c)$ denote the Banach algebra of bounded operators over $c$, the space of convergent sequences. Let $\Gamma$ and $\Delta$ denote the subalgebras of $B(c)$ consisting, respectively, of conservative and conservative triangular infinite matrices, and $C$ the Cesaro matrix of order one. In this paper we investigate Com($C$) in $\Gamma$ and $B(c)$, Com($H$) in $\Gamma$ and $B(c)$ for certain Hausdorff matrices $H$, and some related questions.**

Let $B(c)$ denote the Banach algebra of bounded operators over $c$, the space of convergent sequences. Let $\Gamma$ and $\Delta$ denote the subalgebras of $B(c)$ consisting, respectively, of conservative and conservative triangular infinite matrices. It is well known (see, e.g. [3, p. 77]) that the commutant of $C$, the Cesaro matrix of order one, in $\Delta$ is the family $\mathscr{H}$ of conservative Hausdorff matrices. The same proof yields the result that if $H$ is any conservative Hausdorff triangle with distinct diagonal elements, then Com($H$) $= \mathscr{H}$ in $\Delta$. In this paper we investigate Com($C$) in $\Gamma$ and $B(c)$, Com($H$) in $\Gamma$ and $B(c)$ for certain Hausdorff matrices $H$, and some related questions.

The spaces of bounded, convergent, and absolutely convergent sequences shall be denoted by $m$, $c$, and $l$. $U$ will denote the unilateral shift, and we shall use $A \leftrightarrow B$ to indicate that the operators $A$ and $B$ commute. An infinite matrix $A$ is said to be triangular if it has only zero entries above the main diagonal, and a triangle if it is triangular and has no zeros on the main diagonal. An infinite matrix $A$ is conservative; i.e., $A$: $c \to c$ if and only if

$$\|A\| = \sup_n \sum_k |a_{nk}| < \infty , \qquad a_k = \lim_n a_{nk}$$

exists for each $k$, and $\lim_n \sum_k a_{nk}$ exists.

The proof [2, p. 249] that Com($C$) $= \mathscr{H}$ in $\Delta$, uses the associativity of matrix multiplication. If Com($C$) is to remain unchanged in the larger algebra $\Gamma$, it is necessary that Com($C$) contain only triangular matrices. We are thus led to the following result, where $e_k$ denotes the coordinate sequence with a 1 in the $k$th position and zeros elsewhere.

THEOREM 1. *Let $A$ be a conservative triangle, $B$ an infinite matrix with finite norm, $B \leftrightarrow A$. Then $B$ is triangular if and only if*