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EVALUATION SUBGROUPS OF FACTOR SPACES

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In a series of papers Daniel H. Gottlieb defined and studied evaluation subgroups of homotopy groups. In this paper we develop techniques for calculating these subgroups for some factor spaces. The calculations give information on the vanishing of Whitehead products and the existence of cross sections to certain types of fibrations.

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With the exception of finite topological groups, all spaces are assumed to be locally compact, path connected CW complexes with base point. The base point of spaces $A, B, \cdots, X, Y$ will always be denoted by $a_0, b_0, \cdots, x_0, y_0$. When the domain is clear the symbol x_0 will also denote the constant function with image x_0. 1_A will denote the identity map from A to A for any set A. Homology and cohomology groups are assumed to be singular with integer coefficients. $A \vee B$ and $A \times B$ will denote the one point union and Cartesian products respectively.

The following can be found in [7] or [8] unless otherwise stated.

DEFINITION I.1. The evaluation subgroup $G_n(X)$ is the subgroup of $\pi_n(X)$ containing all elements α which can be represented by a map $f: S^n \rightarrow X$ such that $1_X \vee f: X \times S^n \rightarrow X$ extends to a map $\phi: X \times S^n \rightarrow X$.

The map $\phi: X \times S^n \rightarrow X$ will be called an associated map for $\alpha \in G_n(X)$.

Let M be the path component of the space of maps from X to X containing the identity map. If $\omega: M \rightarrow X$ is the evaluation map defined by $\omega(f) = f(x_0)$, then $G_n(X) = \omega_*(\pi_n(M)) \subset \pi_n(X)$. $G_n(X)$ is then clearly a subgroup. This alternate definition motivated the name evaluation subgroup.

THEOREM I.2. *$G_n(X)$ is the set of all $\alpha \in \pi_n(X)$ such that there is a fibration $p: E \rightarrow S^{n+1}$ with X as a fiber and $\alpha = \partial(\iota_{n+1})$ where $\iota_{n+1} = [1_{S^{n+1}}] \in \pi_{n+1}(S^{n+1})$ and ∂ is the boundary homomorphism in the homotopy exact sequence for p.*

COROLLARY I.3. *If $G_n(X) = 0$, any fibration with base S^{n+1} and fiber X admits a cross section.*

DEFINITION I.4. $P_n(X) \subset \pi_n(X)$ is the set of elements $\alpha \in \pi_n(X)$