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LOCALLY HOLOMORPHIC SETS AND THE LEVI FORM

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Suppose we have a real k-dimensional $\mathscr{C}^2$ manifold M embedded in C^n. If M has a nondegenerate complex tangent bundle of positive rank at some point $p \in M$, then the vanishing or nonvanishing of the Levi form on M near p determines whether or not M is locally holomorphic at p. We show that if M is locally holomorphic at p, then the Levi form vanishes near p, the converse being a known result. In addition we prove a $C-R$ extendibility theorem for a certain case when M is $\mathscr{C}^\infty$ and has a nonzero Levi form at $p \in M$.

1. Introduction. In the study of holomorphic extendibility and holomorphic convexity we often want to know whether a set is a holomorphic set or not. For instance a totally real submanifold of a Stein manifold is a holomorphic set (see [5]). If a real k-dimensional $\mathscr{C}^2$ manifold M is embedded in C^n in such a way that M has a nondegenerate complex tangent bundle at some point p, the property of being locally holomorphic at p depends on the Levi form on M near p. It has been shown that if the Levi form vanishes near p then M is locally holomorphic at p. The converse has been proved only in the generic case when $k > n$ and M is $\mathscr{C}^\infty$ ([1] or [6]). It is the purpose of this paper to prove the converse in all cases. For a particular case (which we call pseudo-hypersurface) we combine a lemma of Nirenberg [4] with the compactly supported solutions to the $C-R$ equations to prove a $C-R$ extension theorem.

In §2 we define exceptional points, the Levi form, and the concept of local holomorphicity. Section 3 contains a discussion of the relation of the Levi form to the local equations of the embedded manifold. In §4 we show that to prove theorems about local holomorphicity, we need only consider open sets of C^n with $\mathscr{C}^\infty$ boundaries. We show that a locally holomorphic set has a vanishing Levi form, if the Levi form can be defined. In §5 we define the concept of a pseudo-hypersurface and prove that if the Levi form does not vanish on a pseudo-hypersurface, all $C-R$ functions are extendible to an open set in C^n.

2. Definitions. Let M be a real k-dimensional $\mathscr{C}^2$ manifold embedded in C^n, $k, n \geqq 2$. Let $T_x(M)$ be the real tangent space to M at x and $H_x(M) = T_x(M) \cap iT_x(M)$. Then $H_x(M)$ is the maximal complex subspace of C^n contained in $T_x(M)$, called the vector space