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COHOMOLOGY IN THE FINITE TOPOLOGY AND BRAUER GROUPS

RAYMOND T. HOOBLER

An exact sequence relating $\mathrm{Br}(X)$, the Brauer group of a regular scheme of dimension $\leqq 2$, and Amitsur cohomology (obtained as the cohomology of the sheaf of units on an appropriate Grothendieck topology) is derived by functorial methods. In order to do this we first show that any torsion element of $H^1(X_{et}, G_m)$, i.e., $\mathrm{Pic}(X)$, and $H^2(X_{et}, G_m)$, i.e., $\mathrm{Br}(X)$, is split by a finite, faithfully flat covering $Y \to X$. After proving a divisibility result for $\mathrm{Pic}(X)$ under such coverings and some preliminary investigation of cohomology in the topology defined from such coverings, the exact sequence which is analogous to that of Chase and Rosenberg is obtained.

Let X be a regular scheme with $\dim X \leqq 2$, i.e. $\mathcal{O}_y$ is a regular local ring for all $y \in X$. Grothendieck has then shown that the Brauer group of the scheme X, $\mathrm{Br}(X)$, is isomorphic to $H^2(X_{et}, G_m)$ where X_{et} is the etale site on X [2]. On the other hand Chase and Rosenberg have given an exact sequence relating the kernel of $\mathrm{Br}(R) \to \mathrm{Br}(S)$ with $\check{H}^2(S/R, G_m)$ where S is a finite, faithfully flat R-algebra [5]. This result suggests that the Brauer group of X, X a regular Japanese scheme with $\dim X \leqq 2$, might be described by $H^2(X_f, G_m)$ where X_f, the finite site on X, is the one suggested by using coverings of the type giving the Chase-Rosenberg exact sequence. Surprisingly, $H^2(X_f, G_m)$ turns out to be too large. The measure of the difference lies in $\mathrm{Pic}(X)$. If $\mathrm{Pic}(X)$ is torsion, then $H^2(X_f, G_m)$ is the Brauer group of X.

Clearly we must first show that any Azumaya algebra on X can be split by a finite, faithfully flat covering of X. This and some curious results on the behaviour of $\mathrm{Pic}(X)$ constitute the major part of the first section. In the next section the cohomology groups, $H^n(X_f, G_m)$, are investigated by spectral sequence arguments, and a sequence similar to the Chase-Rosenberg sequence is derived. The result mentioned above then follows immediately from this sequence and the splitting theorems of the first section. In a forthcoming paper most of these results will be extended to the l-primary component of $\mathrm{Br}(X)$, l a prime, for affine schemes X of characteristic l^n. This accounts for the condition $\mathrm{Sp}(l)$ introduced in the second section.

We have generally adopted the style of Artin's *Grothendieck Topologies* [1] since it seems to be more readily available than SGAA [2]. This makes no difference in the results since all of the topologies