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BOUNDARY RESPECTING MAPS OF 3-MANIFOLDS

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This paper is about maps of compact 3-manifolds which map the boundary of the domain (possibly nonhomeomorphically) into the boundary of the range. F. Waldhausen has shown that such a map between compact, orientable, irreducible 3-manifolds with nonempty, incompressible boundary is homotopic to a homeomorphism if and only if the map induces an isomorphism at the fundamental group level. The main theorem of this paper states that the above theorem remains valid if the assumption of incompressible boundary is dropped.

A study of disk sums of bounded 3-manifolds will be required in order to prove the above-mentioned theorem. This investigation involves theorems about disk sums of bounded 3-manifolds analogous to the classical Kneser theorem for closed 3-manifolds.

The reader may wish to consult [11] for a proof of Waldhausen's theorem mentioned above and [4] and [7] for variations of Waldhausen's theorem related to the theorems proved in this paper.

All spaces and maps in this paper are assumed to belong to the precise linear category, and each subspace that we shall discuss is taken to be piecewise linearly embedded. If A is a subcomplex of the simplicial complex X, we use the notation $U(A, X)$ to denote a regular neighborhood of A in a second derived subdivision of X.

If X is a manifold, we use the notation $\operatorname{bd} X$ and $\operatorname{int} X$ to denote the boundary of X and the interior of X respectively.

A 3-manifold M is said to be *irreducible* if each 2-sphere in M is the boundary of some 3-cell in M.

A compact 2-manifold F embedded in a manifold M is *properly embedded* in M if $F \cap \operatorname{bd} M = \operatorname{bd} F$. A compact 2-manifold F properly embedded in a 3-manifold M is *incompressible* in M if for each disk D in M such that $D \cap F = \operatorname{bd} D$, there exists a disk D' in F such that $\operatorname{bd} D = \operatorname{bd} D'$.

Let F denote a 2-manifold properly embedded in a 3-manifold M, and let J denote a loop in M that meets F transversely. We define the symbol $[J, F]$ to be 0 if J meets F an even number of times, and $[J, F] = 1$ if J meets F an odd number of times. Observe that if J^* is a loop in M that is homotopic to J, then $[J, F] = [J^*, F]$.