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UNIVERSAL COSIMPLE ISOLS

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Our results follow from a single priority scheme which we give in detail. They are (i) if an arbitrary $n+1$-ary relation over the nonnegative integers determines an n-ary function, then its canonical extension to the isols determines a function on the cosimple isols if and only if the function determined on the integers is an almost recursive combinatorial function, and (ii) every countable partially ordered set can be embedded in (a) the cosimple isols, and even more (b) the cosimple regressive isols. The remaining material generalizes and extends these results.

1. Independence. Let $\omega =$ the nonnegative integers, $P =$ the set of all subsets of ω, and $Q =$ the set of all finite subsets of ω. Use $X^k A$ for the k-fold direct power of A. The results of this section all follow from the priority method of [11]. Let $f: \omega \rightarrow X^2 Q$ be a sequence of pairs $f(s) = (\alpha_s, \beta_s)$ where $\alpha_s \cap \beta_s = \varnothing$, and let $g: \omega \rightarrow \omega$. The *requirement* $R_k = \{(\alpha_s, \beta_s) \mid g(s) = k\}$. $\xi \in P$ is said to *meet* the requirement R_k if $\alpha \subseteqq \xi$, $\xi \cap \beta = \varnothing$ for some $(\alpha, \beta) \in R_k$. With every pair (f, g) as above we associate the *priority sequence* $\xi: \omega \rightarrow Q$ constructed in stages. Stage $s = 0$, $\xi_0 = \varnothing$ and for stage $s > 0$, $\xi_s = \xi_{s-1}$ if (1), (2), or (3) below is true. Otherwise $\xi_s = \xi_{s-1} \cup \alpha_s$.

(1) $\xi_{s-1} \cap \beta_s \neq \varnothing$.

(2) there is an $r < s$, $r > 0$, $g(r) < g(s)$ such that

$$\alpha_r \nsubseteqq \xi_{r-1}, \alpha_r \subseteqq \xi_r, \xi_{s-1} \cap \beta_r = \varnothing \text{ and } \alpha_s \cap \beta_r \neq \varnothing .$$

(3) there is an $r < s$, $r > 0$, $g(r) = g(s)$ such that

$$\alpha_r \nsubseteqq \xi_{r-1}, \alpha_r \subseteqq \xi_r \text{ and } \xi_{s-1} \cap \beta_r = \varnothing .$$

The requirement R_k is met at stage s if $s > 0$, $g(s) = k$ and $\alpha_s \nsubseteqq \xi_{s-1}$ but $\alpha_s \subseteqq \xi_s$. The requirement R_k is *injured* at stage s if for some $r < s$, R_k was met at stage r, $\xi_{s-1} \cap \beta_r = \varnothing$ but $\xi_s \cap \beta_r \neq \varnothing$. The basic combinatorial content of the priority method is summarized in the fundamental

LEMMA 1. (Sacks [11]). *For each k the set of stages s where R_k is either met or injured is finite.*

Proof. First we need two facts, (4) and (5) below. They are

(4) if R_k is injured at stage s then for some $j < k$, R_j is met at stage s.