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TRANSFORMATIONS OF SYMMETRIC TENSORS

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This paper is about linear transformations of the k-fold symmetric tensor product of an n-dimensional vector space V which carry nonzero decomposable tensors to nonzero decomposable tensors. The main theorem shows that every such transformation is induced by a nonsingular transformation of V provided both

(i) the field has characteristic either 0 or a prime greater than k and every polynomial over the field with degree at n is a product of linear factors.

(ii) $n > k + 1$.

Condition (i) includes the important special case where the field is algebraically closed with characteristic 0.

The linear transformations which preserve decomposable tensors in the skew-symmetric case have been studied in two papers by Westwick [6, 8]. In [6] he showed that if the field is algebraically closed then the transformation is induced by a linear transformation of V except, possibly, when the dimension of V is $2k$. In the latter case the transformation may be the composition of one induced by a linear transformation of V and one induced by a correlation of the k-dimensional subspaces of V. A series of papers [3, 4, 7, 2] has been devoted to linear transformations which preserve decomposable tensors in the case of the full tensor product.

Our result partially answers a question first raised by Marcus and Newman in [5]. They asked for necessary and sufficient conditions in order that every decomposable mapping of the space of k-fold symmetric tensors be induced.

1. **Preliminaries.** Let V^k denote the k-fold Cartesian product of V where $k > 1$. A *k-fold symmetric tensor space* (or rank k symmetric tensor space) is a vector space denoted by $\bigvee_k V$ together with a fixed multilinear symmetric mapping $\sigma\colon V^k \to \bigvee_k V$ which is universal for multilinear and symmetric mappings of $\bigvee_k V$. We assume that $\bigvee_k V$ is generated by the image of σ. Thus, if W is any vector space and $g\colon V^k \to W$ is both multilinear and symmetric then g has a unique extension $h\colon \bigvee_k V \to W$ such that

(1.1)
$$\begin{array}{ccc} & W & \\ {}^{g}\nearrow & & \nwarrow^{h} \\ V^k & \xrightarrow[\sigma]{} & \bigvee_k V \end{array}$$