PACIFIC JOURNAL OF MATHEMATICS
Vol. 42, No. 3, 1972

GENERALIZED CONTINUATION

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In this paper the operation of analytic continuation is generalized by relaxing the condition that a direct continuation of a function must have the same values as the original on the intersection of their domains of definition. Thus the generalized continuations of a function can have some other property in common with the original function such as being preimages of a single function under a local integral operator. This generalization is accomplished by developing $\mathscr{A}$-continuation of $\mathscr{F} = \{(f_\alpha, S_\alpha) \mid f_\alpha \in \Phi$ and S_α a ball in $\mathscr{C}^n\}$ with respect to a collection of maps, $\mathscr{A}$, of subsets of $\mathscr{F}$ into $\mathscr{F}$. $\mathscr{A}$ must satisfy some compatibility conditions. Many of the proofs in this development parallel those for analytic continuation and lead to the introduction of a manifold on which the generalized continuation is single valued. A generalized continuation of function elements (f_α, S_α) is achieved when all the f_α's are complex valued functions defined on S_α and some examples are given.

In §1 $\mathscr{A}$-continuation is developed for $\mathscr{F}$. A manifold $M(\mathscr{F}, \mathscr{A})$ is developed on which $\mathscr{A}$-continuation is single valued and the complete $\mathscr{A}$-function is introduced which is similar to the complete analytic function of Weierstrass. Theorem 11 states a necessary and sufficient local condition that $M(\mathscr{F}, \mathscr{A})$ and $M(\mathscr{H}, \mathscr{B})$ be holomorphic. In section 2 $\mathscr{A}$-continuation is specialized to sets, $\mathscr{F}$, where f_α is a function with S_α as its domain of definition. Then (f_α, S_α) is referred to as a function element. For function elements a compatible set of maps can be considered as a generalization of direct analytic continuation of power series. An indicator function is defined to help describe a complete $\mathscr{A}$-function. Direct analytic continuation and continuation of the coefficients of a linear Weierstrass polynomial are given as examples.

Given in §3 is the more intricate example of continuing the normalized B_3-associate of the Bergman-Whittaker Integral Operator. Using Theorem 11 this generalized continuation is shown to be equivalent to analytically continuating the harmonic function represented by the B_3-associate. This is the example which motivated the study of generalized continuation.

1. Generalized continuation. Let Φ be a set and with each f_α in Φ associate ball, S_α, in C^n and let $\mathscr{F} = \{(f_\alpha, S_\alpha) \mid f_\alpha \in \Phi\}$. Let x_α denote the center of S_α and consider a set of operators or maps $\mathscr{A} =$