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TOPOLOGIES ON STRUCTURE SPACES OF LATTICE GROUPS

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A structure space of a lattice group G is, coventionally, a set of prime subgroups of G with the hull-kernel topology. The set of all prime subgroups of G, together with G when G has no strong unit, carries a natural topology, stronger than the hull-kernel topology, which is compact and Hausdorff. There is a natural closed subspace which is a quotient of the Stone space of the complete Boolean algebra of polar subgroups. Under the hull-kernel topology this subspace is a retract of the space of prime subgroups, but no longer closed. These topologies are compared, with particular reference to coincidences.

Consideration of structure spaces for lattice groups is not new. Nakano [15], for complete vector lattices and Amemiya [1] for arbitrary vector lattices were the first to treat the question systematically. Indeed the germ of our compactness proof is already in [1] (Theorem 2.1). More recently Isbell [11] and Isbell and Morse [12] have introduced other structure spaces to solve a specific problem in the theory of f-rings.

1. Prime subgroups.

We recall some definitions. Let G be a lattice group written additively, but not necessarily commutative. A subgroup K of G is *solid* if $x \in K$ and $|y| \leqq x$ imply $y \in K$; K is *prime* if K is solid, proper and $x \wedge y = 0$ implies $x \in K$ or $y \in K$ (normality of K is *not* required). Properties of prime subgroups are listed in many places [6, 7, 9, 13]. We record some which will be used later. The solid subgroups containing prime K from a chain under set inclusion and are prime. If K is prime K contains a minimal prime subgroup.

If S is a nonempty subset of G, $S^{\perp} = \{y \in G: |x| \wedge |y| = 0$ for all $x \in S\}$. We have $S \subset S^{\perp\perp}$, $S \subset T$ gives $S^{\perp} \supset T^{\perp}$, $S^{\perp} = S^{\perp\perp\perp}$ and $S^{\perp}$ is a solid subgroup of G. Subgroups M of G such that $M = M^{\perp\perp}$ are called *polar subgroups*. Under set inclusion and $^{\perp}$ for complementation the set of polar subgroups is a complete Boolean algebra [16, 2]. A prime subgroup K is minimal if and only if for each $x \in G$ exactly one of $x^{\perp\perp}$ and $x^{\perp}$ is a subset of K, [9].

A subgroup K of G is a *z-subgroup* if $x^{\perp\perp} \subset K$ for each $x \in K$. This definition makes K solid and is equivalent to the definition given by Bigard [4]. A z-subgroup of G which is prime will be called a