PACIFIC JOURNAL OF MATHEMATICS
Vol. 43, No. 1, 1972

ON LOW DIMENSIONAL MINIMAL SETS

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Let (X, G, f) be a topological transformation group. Suppose that the phase space X is compact, separable metric, and locally contractible and the group G is the additive group of all real numbers R with the usual topology. If X is a minimal set of $\dim_L(X) \leqq 2$ then X is a manifold, imposing a further condition on the action when $\dim_L(X) = 2$. Hence X is a singleton, a circle or a torus according to its dimension.

A topological transformation group is a triple (X, G, f) consisting of a topological space X, a topological group G, and a continuous map f from $G \times X$ into X such that $f(e, x) = x, f(h, f(g, x)) = f(gh, x)$ for any x in X and any g, h in G and the identity element e of G.

The phase space X of a topological transformation group (X, G, f) is called a *minimal set* if for each $x \in X$ the closure of the orbit of x is X itself. A *locally contractible* space X is a space such that for each $x \in X$ and for any open set U containing x there is an open set V containing x, which is contractible in U to the point x.

Chu [3] has shown that if the phase space X is a compact Hausdorff minimal set and $\dim_L(X) \leqq n$, then $H^n(A, L) = 0$ for every proper closed subset A of X under any connected topological group G. Here $\dim_L(X)$ is the cohomology dimension of X in the sense of Cohen ([2], [4]) and L is a principal ideal domain. The Alexander-Spanier cohomology theory is used here. Using this result, Chu has answered questions that were raised by Gottschalk [6]. He proved that the universal curve of Menger and the universal curve of Sierpinski are not minimal sets under any connected topological group.

Chu has also shown that some cohomological natures of a minimal set are similar to those of a generalized manifold. We try to see whether certain minimal sets are actually generalized manifolds. In this regard, we have some results in low dimensions as mentioned in the abstract.

We use the section theorem of Bebutov and Hájek and the umbrella theorem of Bing-Borsuk that we state here.

The section theorem ([11: p. 332] and [8: p. 210])

Given a topological transformation group (X, R, f) with X separable metric and a non-fixed point x_0 in X there exist sections $S \ni x_0$ generating arbitrary small neighborhoods of x_0 in X. If X is locally compact or locally connected, then S may be taken compact or connected respectively. Furthermore, if X is compact and locally connected, then S may be taken locally connected.