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STRONG LIE IDEALS

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R is 2-torsion free semiprime with $2R = R$. A Lie ideal, U, of R-strong if $aua \in U$ for all $a \in R, u \in U$. One shows that U contains a nonzero two-sided ideal of R. If R has an involution, $*$, (with skew-symmetric elements K) a Lie ideal, U, of K is K-strong if $kuk \in U$ for all $k \in K$, $u \in U$. It is shown that if R is simple with characteristic not 2 and either the center, Z, is zero or the dimension of R over the center is greater than 4, then $U = K$. If R is a topological annihilator ring with continuous involution and if U is closed K-strong Lie ideal, $U = C \cap K$ where C is a closed two-sided ideal of R. A Lie ideal, U, of K is HK-strong if $u^3 \in U$ for all $u \in U$. A result similar to the above result for K-strong Lie ideals can be shown. Let R be a simple ring with involution such that $Z = (0)$ or the dimension of R over Z is greater than 4. Let ϕ be a nonzero additive map from R into a ring A such that the subring of A generated by $\{\phi(x): x \in R\}$ is a noncommutative, 2-torsion free prime ring. Suppose $\phi(xy - y^*x^*) = \phi(x)\phi(y) - \phi(y^*)\phi(x^*)$ for all $x, y \in R$. As an application of the above theory, ϕ is shown to be an associative isomorphism.

1. Introduction. R will denote a semiprime ring such that $2R = R$ and if $2r = 0$, then $r = 0$. We call the latter property 2-torsion free. Z will denote the center of R. If R has an involution, $*$, defined on it, S and K will be the set of symmetric and skew-symmetric elements respectively. The Lie and Jordan products are $[x, y] = xy - yx$ and $x \circ y = xy + yx$ for any $x, y \in R$. If $X, Y \subseteqq R$, $[X, Y]$ will denate the additive subgroup generated by the set $\{[x, y]: x \in X \text{ and } y \in Y\}$. An additive subgroup, U, of R is a Lie ideal of R if $[U, R] \subseteqq U$. If R has an involution, we can similarly define a Lie ideal of K.

This paper is concerned with the study of different classes of Lie ideals of both R and K. A Lie ideal, U, of R is said to be R-strong if $aua \in U$ for all $a \in R$, $u \in U$. If U is a Lie ideal of K, U is K-(HK-) strong if $kuk \in U$ ($u^3 \in U$) for all $k \in K, u \in U$.

In the classical theory of the Lie structure of an associative ring, the main theorem [6; Th. 1.3] states: if R is simple and U is a Lie ideal of R, either $U \subseteqq Z$ or $[R, R] \subseteqq U$. We attempt to develop some criteria for differentiating between Lie ideals of R containing $[R, R]$ and R itself. Similar criteria are developed for Lie ideals of K. We