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DISTRIBUTING TENSOR PRODUCT OVER DIRECT PRODUCT

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This paper is an investigation of conditions on a module A under which the natural map

$$A \otimes (\Pi C_\alpha) \longrightarrow \Pi(A \otimes C_\alpha)$$

is an injection. The investigation leads to a theorem that a commutative von Neumann regular ring is self-injective if and only if the natural map

$$(\Pi F_\alpha) \otimes (\Pi G_\beta) \longrightarrow \Pi(F_\alpha \otimes G_\beta)$$

is an injection for all collections $\{F_\alpha\}$ and $\{G_\beta\}$ of free modules. An example is constructed of a commutative ring R for which the natural map

$$R[[s]] \otimes R[[t]] \longrightarrow R[[s, t]]$$

is not an injection.

R denotes a ring with unit, and all R-modules are unital. All tensor products are taken over R.

We state for reference the following theorem of H. Lenzing [2, Satz 1 and Satz 2]:

THEOREM L. (a) *A right R-module A is finitely generated if and only if for any collection $\{C_\alpha\}$ of left R-modules, the natural map $A \otimes \Pi C_\alpha \rightarrow \Pi(A \otimes C_\alpha)$ is surjective.*

(b) *A right R-module A is finitely presented if and only if for any collection $\{C_\alpha\}$ of left R-modules, the natural map $A \otimes \Pi C_\alpha \rightarrow \Pi(A \otimes C_\alpha)$ is an isomorphism.*

THEOREM 1. *For any right R-module A, the following conditions are equivalent:*

(a) *If $\{C_\alpha\}$ is any collection of flat left R-modules, then the natural map $A \otimes \Pi C_\alpha \rightarrow \Pi(A \otimes C_\alpha)$ is an injection.*

(b) *There is a set X of cardinality at least card (R) such that the natural map $A \otimes R^X \rightarrow A^X$ is an injection.*

(c) *If B is any finitely generated submodule of A, then the inclusion $B \rightarrow A$ factors through a finitely presented module.*

Note that condition (c) always holds when R is right noetherian, for then all finitely generated submodules of A are finitely presented.

Proof. (b) $\Rightarrow$ (c): If R is finite, then it is right noetherian and