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AUTOMORPHISMS ON CYLINDRICAL SEMIGROUPS

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This paper characterizes the automorphisms of a cylindrical semigroup S in terms of the automorphisms of the defining subgroups and subsemigroups. The following theorem is representative of the type of information given in this paper.

Let $F: \boldsymbol{R} \rightarrow A$ be a dense homomorphism of the additive real numbers to the compact abelian group A. Let λ be a positive real number. Multiplication by λ shall also denote the automorphism of A whose restriction to $F(\boldsymbol{R})$ is given by $F\lambda F^{-1}$. The set of all such λ for a given F is called Λ_F.

Theorem. Let f and λ be as above. Let G be a compact group. Let

$$S = \{(p, f(p)\, g): p \in \boldsymbol{H} \text{ and } g \in G\} \cup \alpha \times A \times G\ .$$

Then $\alpha: S \rightarrow S$ is an automorphism if and only if $\alpha(p, f(p), g) = (\lambda p, f(\lambda p), \tau(f(p))\xi(g))$; $\alpha(\infty, a, g) = (\infty, \lambda a, \tau(a)\xi(g))$, where $\tau: A \rightarrow G$ is a homomorphism into the centre of G and, $\xi: G \rightarrow G$ is an automorphism. Theorem. Let S be as in theorem above. Let $\mathscr{A}(G)$ be the automorphism group of G, and $Z(G)$, the center of G. The automorphism group of S is isomorphic as an abstract group to $\mathscr{A}(G) \times (\Lambda_F \times \mathrm{Hom}(A, Z(G)))$ with the following multiplication

$$(\xi, (\lambda, \tau))(\bar{\xi}, (\bar{\lambda}, \bar{\tau})) = (\xi \circ \bar{\xi}, (\lambda\bar{\lambda}, (\tau \circ \bar{\lambda})(\xi \circ \bar{\tau})))\ .$$

Cylindrical semigroups play an important role Mislove's description of $Irr(X)$ and are the building blocks used in the construction of a hormos. Hofmann and Mostert [3] have shown that every compact irreducible semigroup is a hormos. The definition and description of a cylindrical semigroup, given in § I, is from their book.

I. Definitions and notation. All spaces are Hausdorff. All homomorphisms are continuous unless otherwise stated. A homomorphism will be called abstract if it is not assumed continuous. A group considered with the discrete topology will be called abstract. A topological semigroup is a topological space, S, together with a continuous associative multiplication $m: S \times S \rightarrow S$; $m(s, t) = st$. All semigroups are topological with identity 1. A topological group is a semigroup with the map $\phi: S \rightarrow S$, $\phi(s) = s^{-1}$, continuous also. An *ideal*, I, in a semigroup, S, is a subset of S such that: if $x \in S$ then $(xI \cup Ix) \subset I$. If S is compact and abelian then S has an ideal $M(S)$ which is minimal with respect to set inclusion, is unique, and is a group. An *idempotent* $x \in S$ has the property $x^2 = x$. The maximal