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GEOMETRIC ASPECTS OF PRIMARY LATTICES

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The incidence structure derived from a primary lattice with a homogeneous basis of three n-cycles is a Hjelmslev plane of level n. A desarguesian Hjelmslev plane $H(R)$ is of level n if and only if R is completely primary and uniserial of rank n.

Introduction. The classical correspondence between vector spaces, projective spaces and complemented modular lattices was extended to finitely generated modules over completely primary and uniserial rings and primary lattices by Baer [5], Inaba [7] and, recently, by Jónsson and Monk [8]. In these extensions, however, an analogue to the classical projective space is missing. It is shown in the present paper, that the appropriate concept is that of a Hjelmslev space as defined by Klingenberg [9], [10] and by Lück [11]. To be correct, this is only shown for the case of a plane geometry, namely Hjelmslev planes of level n, corresponding to primary lattices with homogeneous basis of three n-cycles, and to free modules R^3. Also, we have the complete correspondence only in the desarguesian case. The restriction to this case is justified, as the author believes, by the fact it is well known to be typical for higher dimensional spaces in the classical theory.

In the non desarguesian case, there is a coordinatization theory for Hjelmslev planes of level n given by Drake [6], but this does not seem to lead to a construction of a lattice from the plane. Every primary lattice with a homogeneous basis of three n-cycles, however, leads to a Hjelmslev plane of level n (Theorem 2.13). Planes of level 1 (ordinary projective planes) and of level 2 (uniform Hjelmslev planes) can be shown to be obtainable from lattices. For uniform planes, this was done by the author in [2]. A combination of Theorem 2.13 with results of [4] shows that a desarguesian Hjelmslev plane $\mathscr{H}(\mathscr{R})$ is of level n if and only if $\mathscr{R}$ is completely primary and uniserial of rank n.

0. Definitions.

0.1. Let $\mathscr{H} = (\mathfrak{p}, \mathfrak{G}, I)$ be an incidence structure consisting of a set $\mathfrak{p}$ of points, a set $\mathfrak{G}$ of lines and an incidence relation $I \subseteq \mathfrak{p} \times \mathfrak{G}$. We say that two points p, q of $\mathscr{H}$ are neighbors, $p \sim q$, if there are two different lines G, H such that $p, q\ IG, H$. Neighborhood for lines is defined dually. A mapping $\varphi: \mathscr{H} \rightarrow \mathscr{H}^*$ is a morphism of incidence structures, if it maps points on points, lines on lines and