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THE USE OF MITOTIC ORDINALS IN CARDINAL ARITHMETIC

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In this paper, based on the properties of mitotic ordinals, some results of the cardinal arithmetic are obtained in a rather natural way.

In what follows, any reference to order among ordinal numbers is made with respect to their usual order. Thus, if u and v are ordinals then $u \leqq v$ if and only if $u \subseteqq v$ if and only if "$u \in v$ *or* $u = v$".

DEFINITION. *A nonzero ordinal w is called mitotic if and only if it can be partitioned into $\overline{\overline{w}}$ pairwise disjoint subsets each of type w. Such a partition is called a mitotic partition of w.*

For instance, ω is a mitotic ordinal since ω can be partitioned into denumerably many pairwise disjoint denumerable subsets R_i with $i = 0, 1, 2, \cdots$, where the elements of R_i are precisely the ordinals appearing in the i-th row of the following table:

$$\begin{array}{cccccccc} 0 & 1 & 3 & 6 & \cdot & \cdot & \cdot \\ 2 & 4 & 7 & \cdot & \cdot & \cdot & \cdot \\ 5 & 8 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 9 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & . \end{array}$$

Clearly, each R_i is of type ω.

LEMMA 1. *Let w be a mitotic ordinal. Then w is a limit ordinal. Moreover, for every element S_i of a mitotic partition $(S_i)_{i \in w}$ of w we have:*

$$(1) \qquad \cup S_i = \sup S_i = w \, .$$

Proof. Since S_i is of type w we see that S_i is similar to w. Let f_i be a similarity mapping from w onto S_i. But then by [1, p. 302] we have $x \leqq f_i(x)$ for every $x \in w$. Now, assume on the contrary that w is not a limit ordinal and let k be the last element of w. But then clearly, $k = f_i(k)$ and therefore $k \in S_i$. However, since 1 is not a mitotic ordinal, we see that the mitotic partition of w must have at least two distinct elements, S_0 and S_1. But then $k \in S_0$ and $k \in S_1$ which contradicts the fact that S_0 is disjoint from S_1. Thus, our assumption is false and w is a limit ordinal.