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INTERPOLATION SETS FOR UNIFORM ALGEBRAS

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Let A be a uniform algebra on a compact Hausdorff space X and let $E \subset X$ be a closed subset which is a G_δ. Denote by B_E all functions on $X \backslash E$ which are uniform limits on compact subsets of $X \backslash E$ of bounded sequences from A.

It is proved that a relatively closed subset S of $X \backslash E$ is an interpolation set and an intersection of peak sets for B_E if and only if each compact subset of S has the same property w. r. t. A. In some special cases the interpolation sets for B_E are characterized in a similar way. A method for constructing infinite interpolation sets for A and B_E whenever $x \in E$ is a peak point for A in the closure of $X \backslash \{x\}$, is presented.

With X as above let $S \subset X$ be a topological subspace. Then $C_b(S)$ denotes all bounded continuous complexvalued functions on S and we put $||f|| = \sup \{|f(x)|: x \in S\}$ if $f \in C_b(S)$.

A subset S of $X \backslash E$ closed in the relative topology is called an interpolation set for B_E if any $f \in C_b(S)$ has an extension to $X \backslash E$ which belongs to B_E. If there exists $f \in B_E$ such that $f = 1$ on S and $|f| < 1$ on $(X \backslash E) \backslash S$, we call S a peak set for B_E. If S has both this properties it is called a peak interpolation set for B_E. Peak and interpolation sets for A are defined in the same way.

It is easy to see that B_E is a Banach algebra with the norm $N(f) = \inf \{\sup_n ||f_n||: \{f_n\} \subset A, f_n \rightarrow f$ uniformly on compact subsets of $X \backslash E\}$. It is an interesting problem in itself when this norm coincides with sup norm on $X \backslash E$.

In case $X = \{z: |z| \leqq 1\}$ and A is the classical disc algebra of all continuous functions on X which are analytic in $D = \{z: |z| < 1\}$ the interpolation sets for B_E (where E is a closed subset of ∂X) are characterized by that $S \cap \partial X$ has zero linear measure and that $S \cap D$ is an interpolation set for $H^\infty(D)$, the algebra of all bounded analytic functions on D. This result was obtained in [8] by E. A. Heard and J. H. Wells.

Their work has been generalized in different ways. Various authors have considered more general subsets E of $\{z: |z| \leqq 1\}$ and more general algebras of analytic functions. ([2], [3], [4], [6], [9] and [10]).

In this note we wish to generalize the results of Heard and Wells to the setting of uniform algebras. We start with an extension of Theorem 2 in [8].