PACIFIC JOURNAL OF MATHEMATICS
Vol. 43, No. 2, 1972

WEIGHTED CONVERGENCE IN LENGTH

WILLIAM R. DERRICK

This paper studies the lower semicontinuity of weighted length

$$(*) \qquad \liminf_{n\to\infty} \int_{\gamma_n} f\,ds \geqq \int_{\gamma} f\,ds\,,$$

where the sequence of curves $\{\gamma_n\}$ converges uniformly to the curve γ, and f is a nonnegative lower semicontinuous function. Necessary and sufficient conditions for equality in (*) are obtained, as well as conditions which prevent γ from being rectifiable. Requirements are given for the attainment of the weighted distance, from a point to a set, and the families of functions, for which weighted distance is attained or (*) is satisfied, are shown to be monotone closed from below. Finally, the solutions to the integral inequality

$$(**) \qquad |\gamma(t)-\gamma(0)| \geqq \int_{\gamma[0,t]} f\,ds\,,$$

are shown to be compact if the initial values $\gamma(0)$ lie in a compact set.

Let γ be a curve in Euclidean m-space E^m and f be a real-valued function on E^m. The (f)-weighted length of γ, $\int_\gamma f ds$, has proved of fundamental importance in establishing the path-cut inequality for condensers [2], [3] and the relationship between capacity and extremal length [5], [8]. Theorem (2.4) provides necessary and sufficient conditions for weighted convergence in length, and (2.10) gives conditions under which the weighted distance, from a point to a set, is attained. Corollary (2.6) is a useful special case of [8, Lemma 3.3]. In (3.1) the family of functions, for which weighted distance is attained, is shown to be monotone closed from below, and Theorem (3.2) establishes the compactness of the set of solutions to the contingent equation (**), similar to a result of Filippov [4].

2. Convergence theorems.

NOTATION 2.1. Let E^m denote *Euclidean m-space* consisting of all m-tuples $x = (x_1, \cdots, x_m)$ of real numbers with inner product $\langle x, y\rangle = \sum_{i=1}^m x_i y_i$, for all x, y in E^m and norm $|x| = \langle x, x\rangle^{1/2}$. Throughout this paper, points in E^m will often be denoted by the letters x and y, whereas the letters s, t will be reserved for real numbers. The