PACIFIC JOURNAL OF MATHEMATICS
Vol. 43, No. 3, 1972

AN OBSTRUCTION TO FINDING A FIXED POINT FREE MAP ON A MANIFOLD

MAX K. AGOSTON

The problem of whether a manifold M admits a fixed point free map is an old one. One well known result is that if the Euler characteristic $\chi(M) = 0$, then M has such a map. In the case where M is a closed differentiable manifold this follows easily from the fact that $\chi(M) = 0$ if and only if the tangent bundle of M admits a nonzero cross-section (see Hopf [4]). But $\chi(S^{2n}) = 2$, and S^{2n} certainly admits a fixed point free map, namely, the antipodal map. Therefore, the vanishing of the Euler characteristic of a manifold is only a sufficient, though hardly a necessary, condition for the manifold to have a fixed point free map. In the search for other invariants it is natural to generalize somewhat and state the problem in terms finding coincidence free maps.

The object of this paper is to give an elementary proof of the fact that, given a continuous map $f: (W^n, \partial W^n) \to (M^n, \partial M^n)$ between oriented C^∞-manifolds, there is a well defind obstruction $o(f)$ to finding a special sort of map $F: M \to M$ with the property that $F(x) \neq f(x)$ for all $x \in W$. This is the content of Theorem 1 in §2. F will not necessarily be homotopic to f, but then this is something that should not be required in view of the fact that the antipodal map on S^{2n} is not homotopic to the identity map either. In Theorem 2 we prove that o(identity) behaves naturally with respect to tangential maps.

The author would like to thank the referee for bringing the work of F. B. Fuller ([2] and [3]) and E. Fadell ([1]) to his attention. In §3 we discuss the relationship between this paper and their work and how Theorem 1 might be generalized to the case $f: K^n \to M^n$, where K is an n-complex and M is a 1-connected closed manifold. One of the main differences between our approach and that of Fuller, Fadell, and others who have worked on the question of coincidences of maps is that they have allowed themselves to make changes only by a homotopy. They obtain fairly complete results with that restriction in terms of Lefschetz numbers (see [1]). On the other hand, we partially free ourselves from this requirement (in the sense that our maps will be homotopic only on the $(n-1)$-skeleton in general), so that there are more possibilities for F.

Finally, we have restricted ourselves to differentiable manifolds because all the constructions and proofs, which are quite simple from