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ON A DECOMPOSITION OF TRANSFORMATIONS IN INFINITE MEASURE SPACES

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A decomposition theorem for a measure preserving transformation T on a σ-finite and infinite measure space $(\Omega, \mathscr{B}, m)$ is proved and ergodic theorems are considered.

1. Introduction. A measure preserving transformation T on $(\Omega, \mathscr{B}, m)$ is called of *zero type*, if

$$\lim_{N\to\infty} m(T^{-N}A \cap A) = 0$$

for any measurable set A with $m(A) < \infty$. The transformation T is called of *positive type*, if

$$\limsup_{N\to\infty} m(T^{-N}A \cap A) > 0$$

for any measurable set A with $m(A) > 0$. Krengel and Sucheston [4] showed that every measure preserving transformation can be decomposed into two measure preserving transformations, acting on disjoint invariant measurable sets, such that one of them is of zero type and the other is of positive type. However it seems that, in order to apply this result to ergodic theory, more detailed considerations are necessary. In this paper, we shall improve the result by introducing new concept of positivity and then, applying the obtained result, extend ergodic theorems of Brunel and Keane [1] to infinite measure spaces.

2. The decomposition theorem. A measure preserving transformation T will be called of *weakly positive type*, if T is of positive type and satisfies

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} m(T^{-k}A \cap A) = 0$$

for any measurable set A with $m(A) < \infty$. The transformation T will be called of *strongly positive type*, if T satisfies

$$\limsup_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} m(T^{-k}A \cap A) > 0$$

for any measurable set A with $m(A) > 0$.

THEOREM 1. *If T is a measure preserving transformation on*