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DISCONTINUOUS CHARACTERS AND SUBGROUPS OF FINITE INDEX

H. LEROY PETERSON

For brevity's sake, a subgroup of finite index is called "large." A discontinuous torsion character is (clearly) one whose kernel is a large nonopen subgroup. Compact Abelian groups (and certain other LCA groups) have the same number (either none or infinitely many) of large nonopen subgroups and discontinuous torsion characters. Locally compact Abelian groups of which all large subgroups are open include connected, locally connected, and monothetic (possibly totally disconnected) groups. Contrariwise, there are locally compact groups G which have as many as $2^{|G|}$ large nonopen subgroups. These include nondiscrete torsion Abelian groups of bounded order and all totally disconnected, nonmetrizable, compact groups.

1.1. Notation and discussion. In what follows, everything referred to as a locally compact (or compact) group will be assumed to be Hausdorff and nondiscrete, except when it is necessary to refer to a discrete group or a nondiscrete finite group in the course of a proof. If G is a group, G_d denotes G with the discrete topology. If n is a positive integer, $G^{(n)} = \{x^n : x \in G\}$. The symbol e will universally denote the identity of a group. For two topological groups G_1 and G_2, "$G_1 \cong G_2$" means that G_1 and G_2 are topologically isomorphic. The component of e in a topological group G is denoted by C_G. The *weight* of G, denoted by $w(G)$, is the smallest cardinal number for a basis for the topology of G. The symbol T will denote the *circle group* (of complex numbers with modulus one); a *character* on an Abelian group G is a homomorphism (continuous or not) from G into T. If X is a set, $|X|$ denotes the cardinal number of T.

Vol. I of the treatise of Hewitt and Ross [3] will be used as an encyclopedic reference; however, some elementary facts to be found in any text on topological groups will be assumed and used without comment.

It is evident that the closure of a large subgroup (as defined in the synopsis) of any topological group is always a large open subgroup, and that every open subgroup of a compact group is large. As stated in the synopsis, the kernel of a discontinuous character of finite order is a large nonopen subgroup. More generally, a large subgroup of an Abelian topological group is nonopen if and only if it is contained in the kernel of a discontinuous torsion character. (Equivalently, a finite Abelian topological group has discontinuous characters if and only if the group is not discrete.)